Daily Frequency Zero-Coupon Yield Curve for Government Bonds Traded on e-Bond Trading Platform of the Ghana Fixed Income Market

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Abstract: Even though the Ghanaian government bonds are listed and traded on the e-bond trading platform of the Ghana Fixed Income Market, the secondary bond market of Ghana is still underdeveloped and illiquid. Due to the underdeveloped and illiquid nature of the market, and the accompanying problem of non-availability of enough bond and yield data, the only source of data relied upon for yield curve fitting in Ghana is the primary bond market. Meanwhile, the primary market yield curve has low frequency because it is produced weekly. Furthermore, movement in the primary curve is localized at the short ends. The long ends of the curve remain static for months or even years. This does not help reveal the underlying dynamics of bond yield or bond interest rate in Ghana. Despite the fact that there is scarcity of daily frequency data due to the illiquid and underdeveloped nature of the market, we believe that with appropriate methodology, a practically useful daily frequency yield curve can be produced. This paper seeks to fit the secondary market daily frequency zero-coupon yield curve for the Government bonds traded on the e-bond trading platform, using the piecewise cubic Hermite interpolation. Data is obtained from the Central Securities Depository of Ghana. The results show yield curves which have daily frequency; and reveal the true yield dynamics prevailing in the secondary bond market. The yield curves also show the underlying bond market interest rates resulting from free market operations among market participants, devoid of direct involvement of the central bank.

Key words: e-bond trading platform, Ghana fixed income market, daily zero-coupon yield curve, primary market yield curve.

1. Introduction

The Ghanaian bond market in general has seen some level of improvement over the recent years. In particular, the secondary market has experienced much growth and development over the last few years; especially since the establishment of the Central Securities Depository, Ghana Fixed Income Market and the adoption of the Bloomberg e-bond trading system. The entire bond market capitalization (both government and non-government) increased by 217.95% from 10,348.68 million Ghana cedis (3,943.97 million US dollars) as at the end of 2011, to 32,903.91 million Ghana cedis (12,539.96 million US dollars) as at the end of 2015, which means an average annual growth rate of 33.53%. (The exchange rate was 1.4576 Ghana cedis to 1 US dollar in 2011 and 3.7902 Ghana cedis to 1 US dollar in 2015; we apply an average of 2.6239 Ghana cedis to 1 US dollar for these analyses.) The government bond market capitalization also increased by 218.95% from 9,117.63 million Ghana cedis (3,474.81 million US dollars) as at the end of 2011, to 29,080.90 million Ghana cedis (11,082.98 million US dollars) as at the end of 2015, also resulting in an average annual growth rate of 33.64%. The volume of trade per year has also seen remarkable increase over the years. The volume of trade was 11,859 in 2011. This increased by 284.70%, to 45,622 in 2015, representing an average annual growth rate of 40%. All these figures point to the fact that the Ghana bond market is growing both in terms of size and activity. However, as mentioned earlier, out of the total market capitalization of 32,903.91 million Ghana cedis (12,539.96 million US dollars) as at the end of 2015, an amount of 29,080.90 million Ghana cedis (11,082.98 million US dollars) represents government bond capitalization; meaning 88.38% of the bond market capitalization is made up of government bonds. The rest are corporate bonds, some of which are state-owned institutions' bonds. In effect, just like most other African countries, the Ghanaian bond market has been steadily growing in recent years, but nonetheless remain underdeveloped and illiquid; and is dominated by the government bonds [1], [2].

The yield curve is believed to be an important tool needed in the bond market. For instance, the government yield curve which serves as benchmark yield curve, can be used for pricing corporate bonds [3]. Unfortunately, the Ghana bond market does not have a benchmark secondary market zero coupon yield curve. According to the African Financial Market Initiative (AFMI) of the African Development Bank (AfDB),

> *"Ghana does not have a benchmark yield curve, but the Bank of Ghana and the Ministry of Finance construct a yield curve using yields from primary markets for internal purposes. The auction yields are published on official sites. There is no daily setting of reference yields by the debt management office…With the lack of a secondary market and sporadic yield quotation on Reuters or Bloomberg [e-bond trading platforms], the latest auction yields are considered as a benchmark yield curve. No zero-coupon curve is regularly calculated"* [4], [5].

From the above statement, there are at least, three implications worth considering. First of all, the *secondary bond market* of Ghana doesn't have yield curve (allegedly due to lack of, or sporadic quotation of, yields on the e-bond trading platforms). Even though the Ghana bond market has a yield curve, it is not a secondary market yield curve. Whenever the Bank of Ghana issues Government of Ghana bonds (government bonds) in the primary market, a yield curve is produced to reflect the term structure of interest rates on the day of issue. However, when the securities are subsequently traded in the secondary market, no yield curve is available to reflect the term structure of interest rates in the secondary market. Needless to say, the primary market auction yield curve has some limitations. For instance, since central banks use auction rates to signal interest rates in the economy, the primary market yields are likely to be influenced by Bank of Ghana's assessment of demand conditions; and would not be purely determined by market demand-supply interactions as in the case of secondary market yields [6].

The second implication worth considering is that the bond market of Ghana doesn't have daily yield curve to monitor the movements and dynamics of daily interest rates of bonds. The primary market yield curve (primary curve) being used currently has low frequency. The curve shows interest rates when an auction is done; and thereafter remains unchanged until a subsequent auction takes place in the future. Since 91-day and 182-day treasury bills are normally issued weekly in Ghana, it means the highest frequency for the yield curve is weekly. However, since the longer tenor bonds are issued occasionally and infrequently, it means only the very short ends of the yield curve moves weekly; the longer ends remain static for months and even years. As shown by Fig. 1 and Table 1, only the short end of the primary yield curve shows movements over the selected dates. The long end remains constant over all those periods. This would make the yield curve misleading and not relevant and reliable for timely investment and monetary policy decisions. The yield curve in Ghana has consistently shown 19.75% as yield for 15-year term to maturity for over a year; and would remain so, as long as there is no primary market auction of 15-year bond (similar instance applies to the 10-year and 7-year bonds). Meanwhile, in actual sense, as would be reasonably expected, the underlying yield for 15-year tenor bond has been changing almost every day.

Table 1. Bank of Ghana Primary Market Yields

	3 Mo	6 Mo	1 Yr	2 Yr	3 Yr	5 Yr	7 Yr	10 Yr	15 Yr
Aug -11-2017	0.1262	0.1300	0.1500	0.1700	0.185	0.1825	0.1975	0.1900	0.1975
Aug-18-2017	0.1263	0.1361	0.1500	0.1700	0.185	0.1825	0.1975	0.1900	0.1975
Sep-15-2017	0.1314	0.1403	0.1500	0.1700	0.185	0.1825	0.1975	0.1900	0.1975
Nov-03-2017	0.1316	0.1378	0.1500	0.1700	0.1825	0.1825	0.1975	0.1900	0.1975
Nov-10-2017	0.1333	0.1387	0.1500	0.1724	0.1825	0.1825	0.1975	0.1900	0.1975
Mar-02-2018	0.1337	0.1389	0.1500	0.1650	0.1825	0.165	0.1975	0.1900	0.1975
Mar-09-2018	0.1335	0.1388	0.1500	0.1650	0.1825	0.165	0.1975	0.1900	0.1975

Fig. 1. Bank of Ghana primary market yield curve.

The third obvious implication or observation is that the Ghana bond market does not have zero-coupon yield curve. The existing yield curve is not a zero-coupon yield curve; but rather a primary market yield-to-maturity yield curve. However, central banks and bond markets are expected to be producing estimates of zero-coupon yields [7]. The Central Bank of Ghana, and for that matter the Ghanaian bond market has not met this requirement.

In addition to the above observations about the existing yield curve, according to the AFMI, linear interpolation is the method used in fitting the yield curve being used in Ghana [4], [5]. However, since linear interpolation is not differentiable, the curves would lack smoothness.

The Ghana bond market therefore needs a daily frequency zero-coupon yield curve for the government bonds traded on the Ghana Fixed Income Market. Even though the AFMI observes that there is lack of, or sporadic quotation of, yields and prices of bonds traded on the e-bond trading platform of the Ghana Fixed Income Market, we believe a daily zero-coupon yield curve can be produced for the Ghana bond market if appropriate methodology is used. The purpose of this paper therefore is to produce a secondary market daily frequency zero-coupon yield curve using piecewise cubic Hermite interpolation, for the government bonds traded on the e-bond trading system of the Ghana Fixed Income Market.

2. Data and Methodology

2.1. Daily Bond Data and Data Source

Currently, the government bonds have maturities spanning from 91 days (3 months) to 15 years. After the bonds are issued in the primary market by the Bank of Ghana, they are subsequently traded on the Ghana Fixed Income Market, via the Bloomberg e-bond trading system. Relevant information about the bonds traded each day are accessible at the Central Securities Depository website (www.csd.com.gh). We therefore obtain the required daily data such as the prices, maturity dates, coupon rates of bonds and notes; and discount rates of bills from the Central Securities Depository.

2.2. Filtering of Data

The data is filtered to, as much as possible, exclude any erroneously recorded data item. We also ensure that we use on-the-run bonds, if available, especially with regards to the treasury bills; since they are more than longer tenor bonds, in terms of quantity. For the purpose of this work, we also exclude the 15 year bond since it has call features.

2.3. Extraction of Yields

For the purpose of computing the yields, we use Actual/364 as day-count convention and T+2 as the trade settlement period; per the trade guidelines for the government securities market [8], [9]. To obtain the yields, we use the expression:

$$P(t,t_m) = \sum_{i=1}^{m} \frac{C*F}{(1+R(t,t_i))^{t_i-t}} + \frac{F}{(1+R(t,t_m))^{t_m-t}} \tag{1}$$

P (*t, tm*) = price of the coupon bond at time t, maturing at time *m*
R (*t, ti*) = zero-coupon yield corresponding to *ti* – *t* time period, *i*= 1, 2, 3…*m*
F = face (or par) value of the bond
C = coupon rate of the bond

$$P(t,t_m) = \sum_{i=1}^{m} C*F*\delta(t,t_i) + F*\delta(t,t_m) \tag{2}$$

$$\delta(t,t_i) = (1+R(t,t_i))^{-(t_i-t)} \tag{3}$$

2.4. Curve Fitting and Interpolation of Yields

As said earlier, the linear interpolation method is not differentiable so it produces yield curves which might not be smooth enough. We therefore need to use an interpolation method which is smoother than the linear method. However, due to the fact that the Ghanaian secondary market is not developed and there is lack of enough data (as observed by the AFMI [4], [5]), any method that is excessively smooth can possibly produce a yield curve that oscillates. This is because gaps between observed data points would be very wide and the curve segments between the gabs could overshoot. We therefore adopt piecewise cubic Hermite interpolation which is continuously differentiable *only once* and is shape-preserving [10], [11].

Assuming we have yield data points

$$(X_i, f(X_i), f'(X_i)), \forall i = 0,1\ldots, m$$

Such that

$$X_0 < X_1 < \ldots < X_m$$

We find a function S, such that on each sub-interval (X_i, X_{i+1}), S is a cubic polynomial:

$$S(X) = a(X - X_i)^3 + b(X - X_i)^2 + c(X - X_i) + d \tag{4}$$

The given function f, is interpolated by S, subject to the following conditions:

$$S(X_i) = f(X_i) \tag{5}$$

$$S'(X_i) = f'(X_i) \tag{6}$$

For each (X_i, X_{i+1}), let $h = X_{i+1} - X_i$.
We then solve:

$$d = f(X_i) \tag{7}$$

$$c = f'(X_i) \tag{8}$$

$$b = \frac{3}{h^3}\left[f(X_{i+1}) - f(X_i)\right] - \frac{1}{h}\left[f'(X_{i+1}) + 2f'(X_i)\right] \tag{9}$$

$$a = \frac{1}{h^2}\left[f'(X_{i+1}) + f'(X_i)\right] - \frac{2}{h^3}\left[f(X_{i+1}) - f(X_i)\right] \tag{10}$$

Each cubic polynomial produces a curve. These curves join smoothly to form the entire yield curve. Catmull-Rom method can be used to estimate $f'(X_i)$ as shown here [12].

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_{i-1})}{x_{i+1} - x_{i-1}} \tag{11}$$

At the endpoints,

$$f'(x_0) = \frac{f(x_1) - f(x_0)}{x_1 - x_0} \tag{12}$$

$$f'(x_m) = \frac{f(x_m) - f(x_{m-1})}{x_m - x_{m-1}} \tag{13}$$

For the purpose of illustration, we consider a very simplified example. Let $[X_i, X_{i+1}] = [0, 1]$ be the maturity periods of bonds; $[f(X_i), f(X_{i+1})] = [f(0), f(1)]$ be the corresponding yields of the bonds; and $[f'(X_i), f'(X_{i+1})] = [f'(0), f'(1)]$ be the respective first derivatives estimated with the Catmull-Rom method as specified above. On the interval $[X_i, X_{i+1}]$ we specify a cubic function:

$$S(x) = ax^3 + bx^2 + cx + d \tag{14}$$

We then solve a, b, c and d while satisfying the conditions:

$$S(x_i) = f(x_i) \tag{15}$$

$$S'(x_i) = f'(x_i) \tag{16}$$

$$S(x_{i+1}) = f(x_{i+1}) \tag{17}$$

$$S'(x_{i+1}) = f'(x_{i+1}) \tag{18}$$

$$\begin{bmatrix} x_i^3 & x_i^2 & x_i & 1 \\ 3x_i^2 & 2x_i & 1 & 0 \\ x_{i+1}^3 & x_{i+1}^2 & x_{i+1} & 1 \\ 3x_{i+1}^2 & 2x_{i+1} & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} f(0) \\ f'(0) \\ f(1) \\ f'(1) \end{bmatrix} \tag{19}$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 3 & 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} f(0) \\ f'(0) \\ f(1) \\ f'(1) \end{bmatrix} \tag{20}$$

$$\begin{bmatrix} 2 & 1 & -2 & 1 \\ -3 & -2 & 3 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} f(0) \\ f'(0) \\ f(1) \\ f'(1) \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \tag{21}$$

We therefore have

$$d = f(0) \tag{22}$$

$$c = f'(0) \tag{23}$$

$$\begin{aligned} b &= 3f(1) - 3f(0) - f'(1) - 2f'(0) \\ &= 3\left[f(1) - f(0)\right] - \left[f'(1) + 2f'(0)\right] \end{aligned} \tag{24}$$

$$a = -2f(1) + 2f(0) + f^{'}(1) + f^{'}(0)$$
$$= \left[f^{'}(1) + f^{'}(0) \right] - 2\left[f(1) + f(0) \right] \quad (25)$$

Expressions (22) through (25) are simplified versions of expressions (7) through (10) respectively. Expression (14) which represents the interpolating cubic function can then be written as

$$S(x) = \left(\left[f^{'}(1) + f^{'}(0) \right] - 2\left[f(1) + f(0) \right]\right)x^3 + \left(3\left[f(1) - f(0) \right] - \left[f^{'}(1) + 2f^{'}(0) \right]\right)x^2\left(f^{'}(0) \right)x + f(0)$$
(26)

Or

$$S(x) = \left(\left[f^{'}(1) + f^{'}(0) \right] - 2\left[f(1) + f(0) \right]\right)x^3 + \left(3\left[f(1) - f(0) \right] - \left[f^{'}(1) + 2f^{'}(0) \right]\right)x^2 + \left(f^{'}(0) \right)x + f(0)$$
(27)

By factoring the functional and derivative values out in expression (26) or (27), we have

$$S(x) = f(0)\left(2x^3 - 3x^2 + 1\right) + f(1)\left(-2x^3 + 3x^2\right) + f^{'}(0)\left(x^3 - 2x^2 + x\right) + f^{'}(1)\left(x^3 - x^2\right) \quad (28)$$

Or

$$S\left(x\right) = f(0)\alpha_0\left(x\right) + f(1)\alpha_1\left(x\right) + f^{'}(0)\beta_0\left(x\right) + f^{'}(1)\beta_1\left(x\right) \quad (29)$$

where α_0, α_1, β_0 and β_i are the four *basis functions* which are cubic polynomials.

Cubic polynomials *S(x)* on adjacent intervals join together to form the entire yield curve.

3. Results and Discussion

Fig. 2 shows the modeled secondary market zero-coupon, par and forward yield curves as of 17th March 2017. Fig. 3 also shows the modeled secondary market zero-coupon and par yield curves, as well as the Bank of Ghana's primary market auction yields, as of 17th March 2017. (We selected this date since there was an auction by the Bank of Ghana.) The general yield level depicted by the modeled curves and the Bank of Ghana's curve is almost the same.

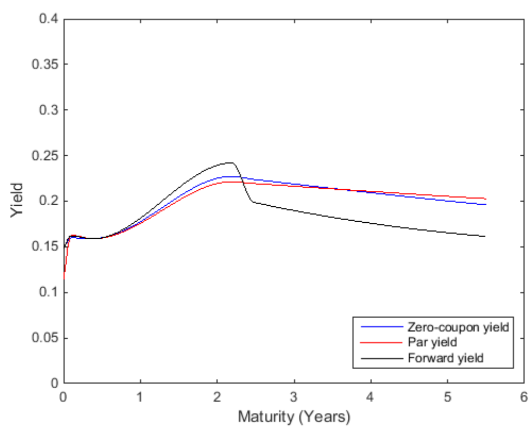

Fig. 2. Modeled yield curves as of 17th March 2017.

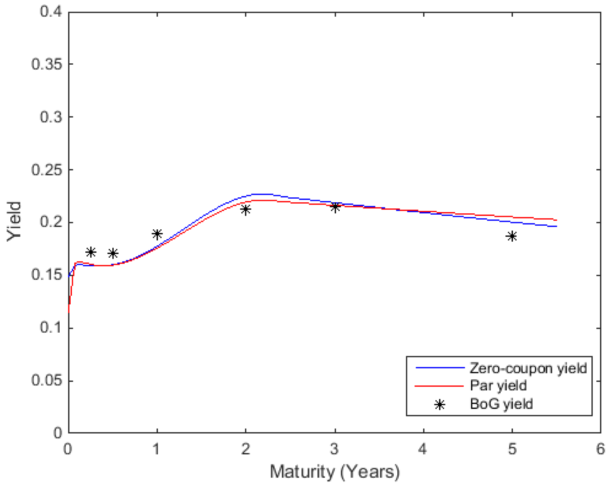

Fig. 3. Modeled yield curves and Bank of Ghana yields as of 17th March 2017.

However, even though the two categories of curves – the modeled curves and the primary curves – show almost the same general yield levels, the modeled curves outperform the primary curves in many ways. First of all, the modeled curves show the market yields determined by pure interaction of demand and supply by the market participants, devoid of direct influence by the Bank of Ghana (unlike the primary yields). Secondly, the modeled curves have daily frequency while the primary yield curve has weekly frequency. That is, while the primary market yield curve is only produced after weekly auctions, the modeled curves are produced after daily secondary market trades. Thirdly, movements in the modeled curve usually apply to the entire maturity spectrum; unlike the existing primary yield curve which usually only varies at the short ends while the longer ends remain unchanged for months or even years (as long as there is no new primary market auction for a particular long term bond). As an example, ever since a 10-year bond was issued in November 2016 at a yield-to-maturity of 19.00%, the weekly yield curve in Ghana has always and constantly been showing 19.00% for 10-year term-to-maturity. This has constantly remained so throughout 2017, and is still so as of April 2018, and would remain so until a 10-year bond is issued in the future (bearing in mind that long term bonds are not issued frequently in Ghana). Meanwhile, our outputs show that the 10-year bond yield (just like other bond yields) varies almost every day (Table 2).

Table 2. Zero-Coupon Yield Curves for 10th – 17th March, 2017

	3 Mo	6 Mo	1 Yr	2 Yr	3 Yr	5 Yr	7 Yr	10 Yr
10th March 2017	0.1576	0.1639	0.1985	0.2024	0.2015	0.1980	0.1922	0.1794
13th March 2017	0.1554	0.1624	0.1943	0.2065	0.2188	0.1843	0.1844	0.1851
14th March 2017	0.1521	0.1551	0.1831	0.2004	0.2196	0.2001	0.1860	0.1749
15th March 2017	0.1558	0.1622	0.1881	0.2074	0.1868	0.1909	0.1910	0.1914
16th March 2017	0.1558	0.1623	0.1891	0.2059	0.2191	0.1966	0.1862	0.1780
17th March 2017	0.1595	0.1599	0.1776	0.2251	0.2188	0.2003	0.1844	0.1688

4. Conclusion and Recommendation

The results show that by using appropriate methodology, secondary market daily zero-coupon yield curve can be produced for the government bonds traded on the e-bond trading platform of the Ghana Fixed

Income Market; even though the market is underdeveloped and illiquid. The results show yield curves which have daily frequency; reveal the yields prevailing in the secondary market; and show the underlying bond market interest rates resulting from free market operations among market participants. Furthermore, the modeled curves show daily yield movements along the entire maturity spectrum of the curve. This curve is more relevant and more reliable for investment and monetary policy decision making; relative to the existing primary market yield curve being used. We recommend that the Ghanaian bond market adopts a suitable method (such as the piecewise cubic Hermite method) to produce secondary market daily zero-coupon yield curve to be used by the market participants for investment, pricing and monetary policy decisions. Since this work considers only the piecewise cubic Hermite method for producing the Ghanaian zero-coupon yield curve, it is recommended that a future work is done to compare the piecewise cubic Hermite method with other popularly used spline based methods and the parametric methods (such as the Nelson-Siegel-Svensson method).

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