On an Integrated Vendor-Buyer Supply Chain Model Subject to Random Breakdowns

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Abstract: We consider an integrated vendor-buyer supply chain production-inventory model with a vendor and a buyer in this paper. The vendor produces and supplies a product to the buyer so that the buyer can meet his demands. The buyer adopts a continuous review and fixed order lot size policy to deal with stochastic demands. All unmet demands are backordered. A lot-for-lot replenishment policy is implemented between the two parties. This implies that upon receiving an order from the buyer, the vendor will immediately start a production run to produce the exact quantity requested by the buyer. The buyer's inventory level is continuously monitored and when it falls to the buyer's reorder point, an order will be placed, and the information is sent immediately to the vendor. A production run will be started at the vendor as soon as the ordering information is received. The vendor's production system is assumed to be subject to random breakdowns. We assume once a breakdown has taken place during a production run, it will require a significant amount of time to perform a correct maintenance activity. Hence, a no-resumption policy is adopted by the vendor. Under this policy, when a breakdown occurs before the desired production lot size is produced, the vendor will order at once the difference between the desired production lot size and the on-hand inventory from external sources. These items must be received by the end of a production run to allow the vendor to ship the planned order quantity to the buyer. An iterative solution procedure is developed to obtain a near-optimal solution for the order quantity and the reorder point.

Key words: Supply chain, stochastic inventory, random machine breakdown.

1. Introduction

A single-manufacturer (vendor) and single-buyer integrated production-inventory model has received a significant amount of attention over the last two decades. Currently, it remains as a popular model because it is a basic building block for studying more complex supply chain problems. Companies in today's competitive market place have recognized that inventories across the entire supply chain must be more efficiently managed through better collaboration and coordination among all parties involved. The benefit will be in a significant reduction of the cost and the lead time without sacrificing customer service. In this paper, an integrated vendor-buyer supply chain production-inventory model with stochastic demands is considered. This supply chain consists of one vendor who produces a product to supply to a buyer. The buyer is facing stochastic demands and is adopting a continuous review, fixed ordering lot size inventory policy, known as a (Q, r) policy, to manage his inventory. Based on this policy, a reorder point r and the order lot size Q are to be determined. Since the inventory is monitored continuously, when the inventory level falls to or below the reorder point level r, an order of Q units will be placed. Upon receiving an order from the buyer, the vendor will immediately start a production run to produce the exact quantity requested by the buyer. After a production runt is completed, the entire lot will be shipped to the buyer. This is called the lot-for-lot policy. During a production run, the demand of the product will be fulfilled by buyer with the inventories kept at the reorder point level. Hence, in this case, the lead time is composed of the production run time, the transportation time, and the non-production time such as production setup time. In order to simplify our subsequent derivations, we assume that the transportation and non-production time are negligible, or they can be prorated into the production time. Since the demand is considered as a random variable, it is conceivable that the demand during the lead time is also a random variable. Therefore, when the lead time demand exceeds the reorder point, shortages will occur. In this paper, we assume all shortages are backordered. Furthermore, the production system is assumed to be subject to random breakdowns. In most single vendor single buyer production-inventory models, the production system is assumed to function flawlessly during the entire production run, and thus no disruptions will occur. In reality, random breakdowns could occur in any production systems. In our model, we consider a case when a production run begins, due to a setup, the production system will be in the normal condition. But as time goes on, machine breakdowns may occur before the end of the production run. When that happens, a time-consuming corrective maintenance activity will be carried out right away. An exponential distribution is used to model the time-to-breakdown random variable. Groenevelt *et al*. [1] investigated the effects of random breakdowns and corrective maintenance on the economic batch sizing decisions. Two inventory control policies were examined in their models, namely, the no-resumption (NR) and abort-resume (AR). The NR control policy assumes that after a breakdown has occurred, the interrupted production is not resumed until all the on-hand inventories are depleted, whereas the AR control policy assumes that if the current on-hand inventory is below a certain threshold value when a breakdown occurs, then this production run is immediately resumed after a corrective maintenance with a negligible repair time is completed. Their research results indicated that this control structure is optimal among all stationary policies and offered the exact optimal and closed form approximate lot sizing formulas and bounds on average cost per unit time for the approximations. In this paper, we consider an integrated vendor-buyer stochastic production-inventory model in which the vendor's production system is subject to random breakdowns. We assume once a breakdown has taken place during a production run, it will require a significant amount of time to perform a correct maintenance activity. Hence, a no-resumption policy is adopted by the vendor. Under this policy, when a breakdown occurs before the planned production lot size is produced, the vendor will order at once the difference between the desired production lot size and the on-hand inventory from external sources. The vendor will have to pay a higher unit variable cost for these items. They must be received by the end of a production run. This will allow the vendor to ship the planned order quantity to the buyer. Our goal is to obtain a solution so that we can achieve a balance between the vendor's costs including the production setup cost, the production cost, the inventory carrying cost, the corrective maintenance cost, and the cost paid to external sources, and the buyer's costs which includes the order setup cost, the inventory carrying cost, and the backorder cost.

Many results can be found in the literature on supply chain inventory models dealing with the integration and collaboration between a vendor and a buyer over a two-level supply chain with either deterministic demands or stochastic demands. Since the problem studied here considers stochastic demands, we only provide a brief review on those papers that dealt with stochastic demands. Aloulou *et al*. [2] gave an extensive review on those results obtained from various stochastic lot-sizing models since year 2000. Pan and Yang [3] considered a single-vendor and single-buyer model in which the lead time is controllable, and the entire order lot size is delivered by multiple shipments. Although Ben-Daya and Hariga [4] studied a similar model, they did take into account a situation where the lead time is a linear function of the production lot size. Moreover, their model included a non-production time as a part of the lead time. Ouyang *et al*. [5] investigated an integrated vendor-buy inventory model with a controllable lead time and stochastic demands. In a separate paper, Ouyang *et al*. [6] extended the model to incorporate the quality improvement cost. Their model assumed that during a production run, the vendor's process may go out of control with a fixed probability each time another item is produced. Once the process is in the out-of-control state, defective items will be produced until the end of the production run. Lin [7] studied an integrated vendor–buyer inventory model with controllable lead time and discounted backorder price discount. Two common investment cost functions to reduce ordering cost are investigated. Chaharsooghi *et al*. [8] proposed a coordination vendor-buyer model with a stochastic demand and lead time. They obtained conditions that can motivate both the vendor and the buyer to participate in a coordination production-inventory policy. Bahri and Tarokh [9] examined a seller-buyer supply chain model with an exponentially distributed lead time. Lin [10] considered a vendor-buyer model with an imperfect production process. A screening process is performed when the buyer receives a shipment. The probability distribution of the demand during lead time is unknown, except the mean and the variance. Vijayashree and Uthayakumar [11] studied a similar model to Ouyang *et al*. [5], but included multiple deliveries from the vendor to the buyer during a production run.

All the studies mentioned above have not considered the vendor's production system that is subject to random breakdowns. As a matter of fact, it was pointed out in Snyder [12] that the effect of random disruptions on the (Q, r) policy is not known. In this paper, we intend to propose a model to fill this existing gap in the literature. The paper is organized as follows: in the next section, we define notation used in this paper and give assumptions for the model. The description of the model is given in Section 3. The derivation of a solution approach is provided in Section 4. Finally, some concluding remarks are given in Section 5.

2. Notation and Assumptions

Before describing our model, we first provide the notation used in this paper as follows:

D = annual demand, a random variable

d = average annual demand, i.e., $E[D] = d$

σ = the standard deviation of the annual demand

p = the vendor's production rate (units/year)

K_2 = the buyer's fixed ordering cost per order

K_1 = the vendor's fixed production setup cost

h_1 = the vendor's cost for unit holding cost per year

h_2 = the buyer's cost for unit holding cost per year

π = unit shortage cost

Q = order quantity (decision variable)

r = reorder point

k = safety factor (decision variable)

L = replenishment lead-time

X = demand during lead time

Y = time-to-breakdown of the vendor's production system

λ = the parameter of the exponentially distributed r. v. Y

a = fixed transportation cost

b = unit variable transportation cost

m = cost of a corrective maintenance

c = vendor's unit production cost

δ = vendor's unit acquisition cost from an external source

The assumptions made in this paper are given as follows:

1) The product is manufactured by the vendor with a finite annual production rate p, and $p > D$.
2) The annual demand is a random variable D with a mean d and a standard deviation σ.
3) The lead time $L(Q)$ is a function of the order quantity Q.
4) The demand during lead time is a random variable with a finite mean $dL(Q)$ and a variance $\sigma^2 L(Q)$.
5) The reorder point r is set equal to the sum of the expected demand during lead time and the safety stock.
6) A continuous-review, fixed order lot size policy is adopted by the buyer.
7) A lot-for-lot production policy is adopted by the vendor.
8) The vendor's production system is subject to random breakdowns during a production run.
9) The shortages are fully backordered by the buyer.
10) The time-to-breakdown is a random variable following an exponential distribution with a mean of $1/\lambda$.
11) The vendor and the buyer have collaborated to derive a solution to minimize an integrated objective
12) function including the vendor's expected annual total cost and the buyer's expected annual total cost.

3. Model Description

The supply chain model studied in this paper includes a vendor who produces a specific product to supply to a buyer, and a buyer who adopts a continuously reviewed, reorder-point, and fixed order lot size policy to fulfill his normally distributed demands with a mean of d units per year. Based on this model, the buyer's inventory level is continuously monitored. Whenever the inventory level falls to the reorder point r, an order of Q unit will be placed by the buyer, and the vendor will receive the ordering information immediately. Upon receiving an order, the vendor then starts a production run with a finite production rate of p, where $p >> D$. Since a lot-for-lot policy is adopted, the vendor will only produce a lot size of Q units. If no breakdowns occur during a production run, then the entire lot will be delivered to the buyer as soon as it is completed. Note that during a production run, the buyer's inventory may continue to decrease due to market demands. However, the buyer has kept in stock r units of the product to meet demands occurring during this period. We assume the transportation time of a production lot is prorated into the production time. This implies that the lead time of the buyers is $L = Q/p$. If the total demand during lead time is higher than the report point, shortages will occur. It is assumed that all shortages are fully backordered and will be filled as soon as the next shipment arrives at the buyer facility. A sample path of the model is depicted in Fig. 1.

In addition, we assume that at the beginning of a production run, a setup will take place that will keep the production system in a normal condition. The production system is subject to random breakdowns during a production run. Therefore, when a breakdown occurs, a corrective maintenance will be carried out right away to restore the system back to the normal condition. However, since the time to complete the corrective maintenance includes the time for the diagnostic process, ordering and waiting time for spare parts, and the time for actual repair work, we assume it is required to spend a significant amount of time to complete the corrective maintenance work. Consequently, not only the vendor will not be able to ship Q units to the buyer by the end of the production run, but also the buyer may have to wait for a long period of time to receive a shipment of Q units from the vendor that will result in a substantial shortages and loss of customer good will. Hence, we assume that the vendor has agreed to deliver a shipment with a size of Q units to the vendor when an order is placed. If a breakdown occurs before the end of a production run, the on-hand inventory level of the vendor will be less than Q. In such a case, according to the agreement, the vendor will have to make up the remaining amount by ordering them from external sources. The vendor must pay for a higher unit variable cost in comparison to the unit production cost when a unit is produced in-house. A sample path of the model for this case is depicted in Figure 2. In summary, our analysis of the model will be divided in two cases depending on whether a breakdown occurs during a production run or not. Next, we will derive the objective function which represents the expected annual total cost for this supply chain model.

Case 1: $Y \geqslant Q/p$

In this case, no breakdowns occur during a production run. Thus, the production run time is $Y = Q/p$, and the lot size of Q will be shipped to the buyer. Because the expected annual demand is d, the length of each cycle for the integrated supply chain is approximated by Q/d. A replenishment order is placed whenever the buyer's inventory level falls to the reorder point r. At the end of the lead time, the order quantity of Q units will arrive at the buyer's facility before the buyer's inventory drops down to zero. However, a total number of demands higher than r may occur during the lead time. As a result, shortages will occur before the arrival of a new order. Since the lead time demand follows a normal distribution, it follows that the lead time demand X has a probability density function $f(x)$ with a finite mean of dL and a standard deviation σ. The reorder point r = expected demand during lead time + safety stock (SS), and can be expressed as follows:

$$r = dL + k\sigma\sqrt{L} = d\left(\frac{Q}{p}\right) + k\sigma\sqrt{\frac{Q}{p}}$$

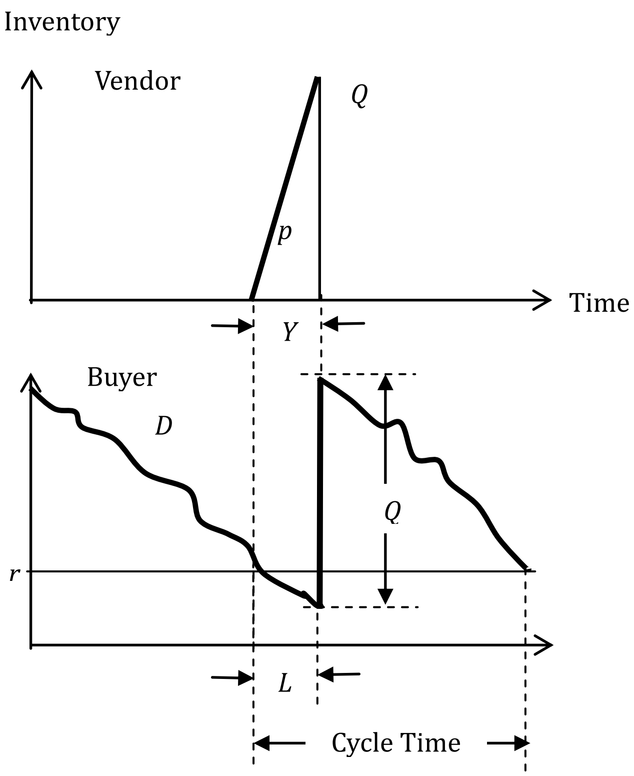

Fig. 1. A sample path of the system for case 1.

In the above equation, k represents the safety factor and satisfies $P(X > r) = P(Z > k) = \theta$, and θ denotes the allowable stock-out probability during the lead time. The safety factor k can be determined from the following equation: $\Phi(k)=\int_k^{\infty}\phi(z)dz$ where $\Phi(k)$ denotes the complementary cumulative distribution function of a standard normal distribution that yields the probability of stock-out during a replenishment cycle at the safety factor *k*, and $\phi(z)$ represents the probability density function of the standard normal random variable *z*. The expected number of units short per cycle is given by

$$E_1[n(r)\,|\,Y=y]=\int_r^{\infty}(x-r)f(x)dx=\sqrt{L}\sigma\Psi(k) \tag{1}$$

where $\Psi(k)$ is the unit linear loss integral and is given as follows:

$$\Psi(k)=\int_k^{\infty}(z-k)\phi(z)dz=\phi(k)-k\Phi(k)\text{, and } k=(r-dL)/\sqrt{L}\sigma$$

Since the demand during lead time is normally distributed, by following the normal approximation, the buyer's expected cost per cycle for this case can be expressed as follows:

$$E_1[TCB\,|\,Y=y]=K_2+\frac{h_2Q}{d}\left(\frac{Q}{2}+r-dL\right)+\pi\sigma\Psi(k)\sqrt{L}=K_2+\frac{h_2Q}{d}\left(\frac{Q}{2}+k\sigma\sqrt{\frac{Q}{p}}\right)+\pi\sigma\Psi(k)\sqrt{\frac{Q}{p}} \tag{2}$$

Next, we calculate the vendor's expected total costs for a cycle. During a production run, the vendor will incur the following costs:

1) the production setup cost is K_1,
2) the inventory holding cost can be computed as $(h_1Q^2)/2p$,
3) the cost for corrective maintenance is 0,
4) the production cost for the product is cQ, and
5) the transportation cost is $a+bQ$.

We can obtain the vendor's expected total cost per cycle as follows:

$$E[TCV_1\,|\,Y=y]=K_1+\frac{h_1}{2p}Q^2+cQ+a+bQ \tag{3}$$

Hence, the expected total cost per cycle for the supply chain can be calculated as follows:

$$\begin{aligned}E_1[TC\,|\,Y=y]&=E_1[TCB\,|\,Y=y]+E_1[TCV\,|\,Y=y]\\&=K_1+K_2+a+\frac{1}{2}\left(\frac{h_1}{p}+\frac{h_2}{d}\right)Q^2+(c+b)Q+\frac{h_2k\sigma Q}{d}\sqrt{\frac{Q}{p}}+\pi\sigma\Psi(k)\sqrt{\frac{Q}{p}}\end{aligned} \tag{4}$$

The expected cycle time of Case 1 can be calculated as follows:

$$E_1[T\,|\,Y=y]=\frac{Q}{d}$$

Case 2: $0 < Y < Q/p$

In this case, a breakdown has occurred before a production run for a lot size of *Q* units is completed. Since only *pY* units is produced, the remaining $Q-pY$ units will be ordered from external sources with a cost of $\$\delta$ per unit which is higher than the regular unit production cost $\$c$ per unit. Note that in this case, only the vendor's cost will be different. First, we calculate the inventory carried by the vendor during a cycle as follows:

$$I=\frac{p}{2}y^2+py\left(\frac{Q}{p}-y\right)=Qy-\frac{p}{2}y^2$$

Then we can obtain the vendor's expected total cost per cycle as follows:

$$E_2[TCV\,|\,Y=y]=K_1+h_1\left[Qy-\frac{p}{2}y^2\right]+m+cpy+\delta(Q-py)+a+bQ$$

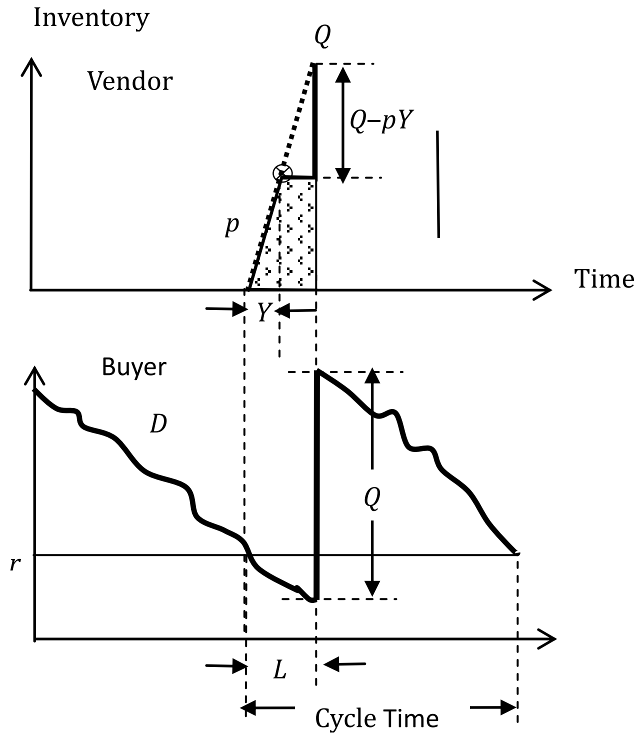

Fig. 2. A sample path of the system for case 2.

The buyer's expected total cost per cycle for this case is identical to the one of Case 1. The expected cycle time of this case is also identical to the one of Case 1. The expected total cost per cycle for this case of the supply chain can be calculated as follows:

$$\begin{aligned}&E_2[TC\,|\,Y=y]=E_2[TCB\,|\,Y=y]+E_2[TCV\,|\,Y=y]\\&=K_1+K_2+a+m+(h_1Q+cp-\delta p)y-\frac{h_1p}{2}y^2+(\delta+b)Q+\frac{h_2}{2d}Q^2+\frac{h_2k\sigma Q}{d}\sqrt{\frac{Q}{p}}+\pi\sigma\Psi(k)\sqrt{\frac{Q}{p}}\end{aligned}\qquad(5)$$

Next, we derive the expression for the objective function of the integrated model. First, we obtain the expected cycle time in year as follows:

$$E[T]=\int_{Q/p}^{\infty}E_1[T\,|\,Y=y]f(y)dy+\int_{0}^{Q/p}E_2[T\,|\,Y=y]f(y)dy=Q/d$$

The expected total cost per cycle can be found as follows:

$$E[TC]=\int_{Q/p}^{\infty}E_1[TC\,|\,Y=y]f(y)dy+\int_{0}^{Q/p}E_2[TC\,|\,Y=y]f(y)dy$$

After some algebra, we obtain the following equation:

$$E[TC]=A+\left(m+\frac{h_1Q+cp}{\lambda}\right)(1-e^{-\lambda Q/p})+\delta\left[Q-\frac{p}{\lambda}(1-e^{-\lambda Q/p})\right]-\frac{h_1p}{\lambda^2}\left(1-e^{-\lambda Q/p}-\frac{\lambda Q}{p}e^{-\lambda Q/p}\right)+bQ$$
$$+\frac{h_2}{2d}Q^2+\frac{h_2k\sigma Q}{d}\sqrt{\frac{Q}{p}}+\pi\sigma\Psi(k)\sqrt{\frac{Q}{p}} \quad (6)$$

where $A = K_1 + K_2 + a$.

The expected annual total cost can now be computed as follows:

$$\frac{E[TC]}{E[T]}=\frac{Ad}{Q}+\left(m+\frac{cp}{\lambda}\right)\frac{d}{Q}(1-e^{-\lambda Q/p})+\frac{h_1d}{\lambda}(1-e^{-\lambda Q/p})+\delta d\left[1-\frac{p}{\lambda Q}(1-e^{-\lambda Q/p})\right]$$
$$-\frac{h_1pd}{\lambda^2Q}\left(1-e^{-\lambda Q/p}-\frac{\lambda Q}{p}e^{-\lambda Q/p}\right)+bd+\frac{h_2}{2}Q+h_2k\sigma\sqrt{\frac{Q}{p}}+\frac{\pi d\sigma\Psi(k)}{Q}\sqrt{\frac{Q}{p}} \quad (7)$$

After excluding the cost components not involving the decision variable Q, we can express the expected annual relevant cost as follows:

$$Z(Q,k)=\frac{Ad}{Q}+\left(m+\frac{cp}{\lambda}\right)\frac{d}{Q}(1-e^{-\lambda Q/p})+\frac{h_1d}{\lambda}(1-e^{-\lambda Q/p})+\delta d\left[1-\frac{p}{\lambda Q}(1-e^{-\lambda Q/p})\right]$$
$$-\frac{h_1pd}{\lambda^2Q}\left(1-e^{-\lambda Q/p}-\frac{\lambda Q}{p}e^{-\lambda Q/p}\right)+\frac{h_2}{2}Q+h_2k\sigma\sqrt{\frac{Q}{p}}+\frac{\pi d\sigma\Psi(k)}{Q}\sqrt{\frac{Q}{p}} \quad (8)$$

We can rewrite $Z(Q, k)$ as follows:

$$Z(Q,k)=\Omega_1+\Omega_2+\Omega_3+\Omega_4+\Omega_5$$

where

$$\Omega_1=\frac{Ad}{Q}$$

$$\Omega_2=\left(m+\frac{cp}{\lambda}\right)\frac{d}{Q}(1-e^{-\lambda Q/p})$$

$$\Omega_3=\frac{h_1d}{\lambda}(1-e^{-\lambda Q/p})-\frac{h_1pd}{\lambda^2Q}\left(1-e^{-\lambda Q/p}-\frac{\lambda Q}{p}e^{-\lambda Q/p}\right)$$

$$\Omega_4=\delta d\left[1-\frac{p}{\lambda Q}(1-e^{-\lambda Q/p})\right]$$

$$\Omega_5=\frac{h_2}{2}Q+h_2k\sigma\sqrt{\frac{Q}{p}}+\frac{\pi d\sigma\Psi(k)}{Q}\sqrt{\frac{Q}{p}}$$

If we examine the Hessian matrix for Ω_i, i=1, ... 5, with respect to Q and k, we can discover that some are positive and some are negative. This implies that the objection $Z(Q, k)$ is neither convex nor concave. The goal is to find a solution for Q, and k so that the objective function $Z(Q, k)$ is minimized. In most real-life cases, often $\lambda Q/p$ is rather small. This allows us to use a Maclaurin series to approximate all exponential terms as follows:

$$e^{-\lambda Q/p} \cong 1 - \frac{\lambda Q}{p} + \frac{1}{2}\left(\frac{\lambda Q}{p}\right)^2$$

After applying this approximation in (8), we obtain a new objection function as follows:

$$Z(Q,k) = \frac{Ad}{Q} + \frac{d}{2p}\left[h_1 + \frac{h_2 p}{d} - (\delta - c)\lambda - \frac{m\lambda^2}{p}\right]Q + h_2 k\sigma\sqrt{\frac{Q}{p}} + \frac{\pi d\sigma\Psi(k)}{Q}\sqrt{\frac{Q}{p}} + \left(m + \frac{cp}{\lambda}\right)\frac{\lambda d}{p} \tag{9}$$

The first partial derivatives of the function $Z(Q, k)$ are given as follows:

$$\frac{\partial Z}{\partial Q} = -\frac{dA}{Q^2} + \frac{d}{2p}\left[h_1 + \frac{h_2 p}{d} + \left(\delta - c\right)\lambda - \frac{m\lambda^2}{p}\right] + \frac{h_2 k\sigma}{2\sqrt{p}}Q^{-\frac{1}{2}} - \frac{\pi d\sigma\Psi(k)}{2\sqrt{p}}Q^{-\frac{3}{2}} \tag{10}$$

$$\frac{\partial Z}{\partial k} = \frac{h_2\sigma}{\sqrt{p}}Q^{\frac{1}{2}} - \frac{\pi d\sigma\Phi(k)}{\sqrt{pQ}} \tag{11}$$

In the next section, we provide some properties which allow us to obtain a near-optimal solution for Q and k that minimize the objection function $Z(Q, k)$.

4. Solution Method

Since it is not easy to show the convexity of $Z(Q, k)$, we will proceed to find some properties that allow us to obtain a near-optimal solution for Q and k that minimize $Z(Q, k)$. First, we show that for a given value of k, an optimal solution of Q can be found despite the fact that the objective function $Z(Q, k)$ is not a convex function of Q.

Property 1. For fixed values of k, the expected annual relevant cost function $Z(Q, k)$ is not a convex function of Q. However, an optimal solution of Q can be determined uniquely.

Proof. The second-order partial derivative of Z with respect to Q can be obtained as follows:

$$\frac{\partial^2 Z}{\partial Q^2} = \frac{2dA}{Q^3} - \frac{h_2 k\sigma}{4\sqrt{p}}Q^{-\frac{3}{2}} + \frac{3\pi d\sigma\Psi(k)}{4\sqrt{p}}Q^{-\frac{5}{2}} = Q^{-\frac{3}{2}}\left(\frac{2dA}{Q^{3/2}} - \frac{h_2 k\sigma}{4\sqrt{p}} + \frac{3\pi d\sigma\Psi(k)}{4\sqrt{pQ}}\right)$$

For a large value of Q, the first and the third term inside the parentheses of the above equation converge to zero. Thus, we can obtain that $\frac{\partial^2 Z}{\partial Q^2} < 0$ for a large value of Q. In addition, we can show that $\lim_{Q\to\infty}\frac{\partial^2 Z}{\partial Q^2} = 0$. This implies that $Z(Q, k)$ is not a convex function of Q. Next, we show that a solution exists. Equation (9) can be rewritten as follows:

$$\frac{\partial Z}{\partial Q} = \frac{d}{2p}\left[h_1 + \frac{h_2 p}{d} + \left(\delta - c\right)\lambda - \frac{m\lambda^2}{p}\right] + Q^{-\frac{1}{2}}\left(\frac{h_2 k\sigma}{2\sqrt{p}} - \frac{dA}{Q^{3/2}} - \frac{\pi d\sigma\Psi(k)}{2\sqrt{p}}Q^{-1}\right) \tag{12}$$

The last three terms in the bracket of the above equation converges to zero for a large value of Q. This implies that

$\lim_{Q\to\infty}\frac{\partial Z}{\partial Q}=\frac{d}{2p}\left[h_1+\frac{h_2 p}{d}+(\delta-c)\lambda-\frac{m\lambda^2}{p}\right]>0$ for $h_1+\frac{h_2 p}{d}+(\delta-c)\lambda>\frac{m\lambda^2}{p}$. Note if this condition is not true, it implies that $m>p\left[h_1+h_2 p/d+(\delta-c)\lambda\right]/\lambda^2$. Under such a circumstance, the vendor may choose not to produce the product in-house to avoid a huge cost for taking the corrective maintenance activity. Next, we examine $\partial Z/\partial Q$ when Q = 1. We first obtain

$$\left.\frac{\partial Z}{\partial Q}\right|_{Q=1}=\frac{d}{2p}\left[h_1+\frac{h_2 p}{d}+(\delta-c)\lambda-\frac{m\lambda^2}{p}\right]+\frac{h_2 k\sigma}{2\sqrt{p}}-dA-\frac{\pi d\sigma\Psi(k)}{2\sqrt{p}}$$

On one hand, if $\left.\frac{\partial Z}{\partial Q}\right|_{Q=1}\geq 0$, then $\frac{\partial Z}{\partial Q}\geq 0$ for all Q. This implies that $Z(Q, k)$ is a strictly increasing function of Q. The optimal solution of Q can be selected as a minimum possible lot size of Q, say, Q_{min}. On the other hand, if $\left.\frac{\partial Z}{\partial Q}\right|_{Q=1}<0$, then the first-order partial derivative $\partial Z/\partial Q$ changes from negative to positive only once in the interval $1 \leq Q < \infty$. Hence, a unique solution of the equation $\partial Z/\partial Q = 0$ exits. Since the derivative $\partial Z/\partial Q$ denotes the slope of the function at Q under a give value of k, it is clear that the function $Z(Q, k)$ is convex. Hence, an optimal solution can be found.

Lemma 2. For a given value of Q, the expected annual relevant cost function $Z(Q, k)$ is a convex function of k.

Proof. This can be shown from the second partial derivative of $Z(Q, k)$ with respect to k as follows:

$$\frac{\partial^2 Z}{\partial k^2}=\frac{\pi d\sigma\phi(k)}{2\sqrt{pQ}}\geq 0\text{, for all } k$$

Next, by using these properties, we develop an iterative procedure that is similar to the one used often in the literature for the classical continuous review (Q, r) problem. Frist, we obtain two equations for finding Q and k by solving $\partial Z/\partial Q = 0$ and $\partial Z/\partial k = 0$. This yields the following two equations

$$Q=\sqrt{\frac{2d\left(A+\frac{\pi\sigma\Psi(k)\sqrt{L(Q)}}{2p}\right)}{\frac{d}{p}\left[h_1+\frac{h_2 p}{d}+(\delta-c)\lambda-\frac{m\lambda^2}{p}\right]+\frac{h_2 k\sigma}{pL(Q)}}} \tag{13}$$

$$\Phi(k)=\frac{h_2 Q}{\pi d} \tag{14}$$

Now, we are ready to develop a solution procedure to obtain a near-optimal solution (Q^*, k^*) for minimizing the expected annual relevant cost $Z(Q, k)$.

Algorithm 1

Step 1: Initialization Set

Set i = 0. Set $Q^{(0)}$ = EOQ. If $Q^{(0)} < Q_{\min}$, set $Q^{(0)} = Q_{\min}$. Use (14) with $Q^{(0)}$ to compute $k^{(0)}$.

Step 2: Set $i = i + 1$. Check the sign of $\partial Z/\partial Q$ at $Q = 1$ using $k^{(i-1)}$. If the sign is non-negative, set $Q^{(i)} = Q_{\min}$. Otherwise, use (13) with $Q^{(i-1)}$ and $k^{(i-1)}$ to compute Q'. If $Q' < Q_{\min}$, then $Q^{(i)} = Q_{\min}$; otherwise, $Q^{(i)} = Q'$. Let ε_1 denote a small tolerance. If $\left|Q^{(i)} - Q^{(i-1)}\right| < \varepsilon_1$, go to Step 4.

Step 3: Use (14) with $Q^{(i)}$ to compute $k^{(i)}$. Let ε_2 denote a small tolerance. If $\left|k^{(i)} - k^{(i-1)}\right| < \varepsilon_2$, go to step 5; otherwise, return to Step 2.

Step 4: Set $Q^{*}= Q^{(i)}$ and $k^{*}= k^{(i-1)}$ Stop.

Step 5: Set $Q^{*}= Q^{(i)}$ and $k^{*}= k^{(i)}$. Stop.

5. Concluding Remarks

This paper considers an integrated vendor-buyer supply chain production-inventory model where a vendor supplies a product to a buyer. The buyer adopts a continuous review and fixed order lot size policy to deal with normally distributed demands. All shortages are backordered. A lot-for-lot replenishment policy is adopted between the two parties. The buyer's inventory level is continuously monitored. Once the inventory level is falling to or under the buyer's reorder point, an order will be placed, and the information is sent to the vendor immediately. Upon receiving an order, the vendor will start a production run. The vendor's production system is subject to random breakdowns. We assume once a breakdown has occurred during a production run, it will require a significant amount of time to perform a correct maintenance activity. Therefore, a no-resumption policy is adopted by the vendor. Under this policy, when a breakdown takes place before the desired production lot size is produced, the vendor will immediately order the difference between the desired production lot size and the on-hand inventory from external sources. These units will be received by the end of a production run so that the vendor can ship the planned order quantity to the buyer. An iterative solution procedure is developed to obtain a near-optimal solution for order quantity and the reorder point. In this paper, the demand distribution is assumed to follow a normal distribution. As a future research direction, we can consider a case where the demand distribution is unknown but with a known mean and variance. In addition, we can study a case in which investment can be made to improve the reliability of production system. Furthermore, it is worth investigating a situation where the corrective maintenance is not time-consuming. Finally, we may also consider a case where defective items may be produced during a production run along with random breakdowns.

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