Mathematical Models for Social Impacts of the Modern Technology Developments

Yixun Shi

Abstract—**Rapid developments of modern technology have impacted the lives of all peoples. In this paper, we discuss a few mathematical models for estimating social impacts of such developments on today's society. In particular, we consider the impacts in the area of communication. Mathematical models for estimating those impacts are discussed, numerical procedures to implement these models are mentioned, and simulated data are used to illustrate the applications of these models.**

Index Terms—**Technology development, social impact, communication, mathematical models.**

I. Introduction

Today's world is in the era of rapid developments of modern technologies. New ideas, new designs, new applications, and new devices pop up every day. The impacts of such developments on our societies are broad and huge. The technology developments have changed our lives in many aspects. For example, how many people are still regularly writing letters to their families and friends today? Not many! At some point in our lives, emails and phone calls were replacing letters. Now, text messaging, online chatting, and social web (such as Facebook) posting have started replacing emails and phone calls.

In this paper, we discuss a few mathematical models for estimating the social impacts of technology developments in the area of communication. Markov chain models are used to estimate the long term trend of the impacts in those areas, entropy models are used to estimate the diversity of the impacts on different groups of people, and χ^2 tests are used to estimate the homogeneity of impacts on different groups of people. Numerical procedures to implement these models are mentioned and simulated data are used to illustrate the applications of these models. These models are ready to be applied by sociologists once they have real world data available.

II. Markov Chain Models for Long Term Trends

In this section, we consider the long term trends of the social impacts of technology developments in the areas of communication and education. In a later section, we will use a dynamical system model to estimate the trend of the impacts in the career options.

Take the area of communication to start with. Let's consider three basic ways of communication: by regular mail, by email or telephone, by text messaging, online chatting, and social web posting. Let us categorize people in three groups: Group A contains those who still use regular mails a lot (either for social purposes or for paying bills) and also use emails and phones a lot, but barely use the newest technology such as text messaging, online chatting, or social web posting; Group B contains those who rarely use regular mails for social purposes but only for paying bills. They mainly use emails and phone calls for social communications, and they have started to try the text messaging, online chatting and social web posting; Group C contains those who are "tech advanced". They rarely use regular mails now. Sometimes they use emails and telephones, but they prefer to use the newest online technology for all the social communications, plus paying their bills. When time goes by, the technology is getting more and more affordable and user-friendly, and accordingly we see more and more people leaning to the use of technology.

So, what would happen in long run? Would we ever need post offices in the future? Let us discuss the use of Markov chain models in estimating the long term trend.

Let us start illustrating this model with an example. Suppose a survey is made at the beginning of each year to find out the percentages of our population in each of the three groups. Each survey will show a status vector $S = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ where x_1, x_2, and x_3 stands for the percent of the population in groups *A, B,* and *C,* respectively. Clearly $0 \le x_1, x_2, x_3 \le 1$ and $x_1 + x_2 + x_3 = 1$. To be more precise, let us use S_1 to denote the initial status vector, that is, the status vector at the beginning of the first year, S_2 to denote the status vector at the beginning of the second year, and so on so forth.

Suppose the survey at the beginning of the first year shows that the initial status $S_1 = \begin{bmatrix} 0.10 \\ 0.60 \\ 0.30 \end{bmatrix}$, that is, at that time 10% of the population are in Group A, 60% in Group B, and 30% in Group C. Suppose the succeeding surveys show that in each year, 10% of those in Group A move to B, 90% of those in group A stay in A, and none of them jump into Group C. For those in Group B, none would move to A, 90% stay in B, and 10% move to C. For the people in Group C, none goes to A, 3% move to B for various reasons, and 97% stay in A.

Based on these hypothetic assumptions regarding the changes in the trend, we can see that

Manuscript received April 10, 2012; revised July 5, 2012.

Yixun Shi is with the Department of Mathematics, Computer Science and Statistics Bloomsburg University of Pennsylvania, USA (yshi@bloomu.edu).

DOI: 10.7763/IJEEEE.2012.V2.134

$$S_2 = \begin{bmatrix} 0.90 & 0 & 0 \\ 0.10 & 0.90 & 0.03 \\ 0 & 0.10 & 0.97 \end{bmatrix} S_1 = \begin{bmatrix} 0.90 & 0 & 0 \\ 0.10 & 0.90 & 0.03 \\ 0 & 0.10 & 0.97 \end{bmatrix} \begin{bmatrix} 0.10 \\ 0.60 \\ 0.30 \end{bmatrix} = \begin{bmatrix} 0.09 \\ 0.559 \\ 0.351 \end{bmatrix} \quad (1)$$

That is, after one year 9% of the population will be in Group A, about 59% in Group B, and about 35% in Group C. Similarly we have

$$S_3 = \begin{bmatrix} 0.90 & 0 & 0 \\ 0.10 & 0.90 & 0.03 \\ 0 & 0.10 & 0.97 \end{bmatrix} S_2 = \begin{bmatrix} 0.90 & 0 & 0 \\ 0.10 & 0.90 & 0.03 \\ 0 & 0.10 & 0.97 \end{bmatrix} \begin{bmatrix} 0.09 \\ 0.559 \\ 0.351 \end{bmatrix} = \begin{bmatrix} 0.08100 \\ 0.52263 \\ 0.39637 \end{bmatrix} \quad (2)$$

So after two years about 8% of the population will be in Group A, about 52% in Group B, and about 40% in Group C.

If a simple estimate is made based on the surveys of those few years and we believe that the same changes will happen in every year in long run, then a Markov chain $S_1, S_2, S_3 \dots$ is formed, with the transition matrix

$$P = \begin{bmatrix} 0.90 & 0 & 0 \\ 0.10 & 0.90 & 0.03 \\ 0 & 0.10 & 0.97 \end{bmatrix} \quad (3)$$

It is easy to see that this is a regular Markov chain with the stationary status vector

$$S = \lim_{k\to\infty} S_k = \begin{bmatrix} 0 \\ 0.23077 \\ 0.76923 \end{bmatrix} \quad (4)$$

In other words, if the same changes keep going on, then eventually regular mails through post offices will only be needed for paying bills and rare use of social purposes.

Note that the stationary status S in equation (4) satisfies

$$PS = S \quad (5)$$

It is also well known that for a regular Markov chain like this, the stationary status S does not depend on the initial status vector S_1. That is, for any initial status S_1, under the constant transition matrix $P = \begin{bmatrix} 0.90 & 0 & 0 \\ 0.10 & 0.90 & 0.03 \\ 0 & 0.10 & 0.97 \end{bmatrix}$, the stationary status will always equal $\begin{bmatrix} 0 \\ 0.23077 \\ 0.76923 \end{bmatrix}$. Furthermore, the transition matrix P may change from year to year. Therefore, the above model may be further generalized into a Markov chain model with changing transition matrices. More precisely, we may use

$P(k) = \begin{bmatrix} p_{11}(k) & p_{12}(k) & p_{13}(k) \\ p_{21}(k) & p_{22}(k) & p_{23}(k) \\ p_{31}(k) & p_{32}(k) & p_{33}(k) \end{bmatrix}$ to denote the transition matrix for the k-th year, with

$$0 \le p_{ij}(k) \le 1, \text{ for all } i, j = 1, 2, 3 \quad (6)$$

and

$$p_{11}(k) + p_{21}(k) + p_{31}(k) = 1,$$
$$p_{12}(k) + p_{22}(k) + p_{32}(k) = 1,$$
$$p_{13}(k) + p_{23}(k) + p_{33}(k) = 1 \quad (7)$$

Many classes of functions may be applied to formulate the transition matrices $P(k)$ based on various social assumptions. For further details in selecting function classes, estimating parameters involved in the functions, and maintaining the models, see [4] and [5].

III. Entropy Models for Diversity of the Impacts

What groups of our society are most impacted by the rapid development of technology, in terms of ways of communication? For example, if we divide our population into classes by age (young, middle age, elderly) and gender (male, female), then we have six classes: male-young, male-middle age, male-elderly, female-young, female-middle age, and female-elderly. Let $p_1, p_2 \dots p_6$ stand for the percent of each class in our population. The entropy model may be applied to measure the diversity of the population.

$$E = -\sum p_i \ln(p_i)$$
$$\text{with } p_1, p_2 \dots p_6 \ge 0 \text{ and } \sum p_i = 1 \quad (8)$$

The value of E reaches its maximum when the population reaches the maximum diversity with $p_1 = p_2 = \dots = p_6$. The value of the entropy E will become smaller if the population is less diverse. For example, if $p_1 = p_2 = \dots = p_6 = 1/6$, then the value of entropy $E = 1.792$. If the population diversity changes a little, say $p_1 = p_2 = p_3 = p_4 = 0.2$ and $p_5 = p_6 = 0.1$, then $E = 1.748$. In case $p_1 = 0.9$ and $p_2 = p_3 = p_4 = p_5 = p_6 = 0.02$, then the value of E goes down to 0.486.

We then may look into the diversity estimates of groups A, B and C defined in the previous section. Let $a_1, a_2 \dots a_6$, $b_1, b_2 \dots b_6$, and $c_1, c_2 \dots c_6$ stand for the percent of each class in groups A, B and C, respectively. Then we may compute the entropy for each of the three groups. By comparing the entropies of the entire population and of the three groups, we may estimate the diversity of the impacts on different classes of people.

TABLE I: Comparison of Entropies

Classes	M-young	M-middle	M-elderly	F-young	F-middle	F-elderly	Entropy
Population	0.15	0.20	0.15	0.15	0.20	0.15	1.782
Group A	0.01	0.10	0.35	0.01	0.13	0.40	1.32
Group B	0.05	0.35	0.10	0.05	0.38	0.07	1.451
Group C	0.35	0.13	0.02	0.40	0.09	0.01	1.34

Let us use a simulated data to illustrate the application of the entropy model. The data and the entropy values are given in the following Table 1. The results clearly show that, in this simulated society, the developments of technology have made a significant difference in diversity in the way people communicate.

$$E(k) = \begin{bmatrix} EP(k) \\ EA(k) \\ EB(k) \\ EC(k) \end{bmatrix}, k = 1, 2, \ldots \quad (9)$$

will form a dynamical system, and various data fitting models may be applied to estimate the long term trend of $E(k)$. See [2] and [3], for example.

IV. χ^2 Tests for Homogeneity of Impacts on Different Groups of People

Do technology developments have equal impacts on peoples in different nations? In different cultures? At different income levels? Or with different education backgrounds?

The χ^2 test may be used to estimate the homogeneity of impacts on different groups of people, in the area of communication.

For example, suppose we consider peoples in a certain nation, and divide them into three groups by the income levels: Lower Income, Middle Class, and Wealthy. We like to see whether these groups have the same proportion distributions in the groups A, B, and C defined earlier. In other words, we like to see whether the technology developments have equal impacts on those three groups of people. χ^2 test may be applied to find that out.

TABLE II: Sample Proportion Distribution

Income\Technology	Group A	Group B	Group C	Total
Lower Income	10	80	10	100
Middle Class	10	100	40	150
Wealthy	10	20	20	50
Total	30	200	70	300

Let us use a simulated data to illustrate the application of this method. Suppose a random sample of 300 people in this nation were interviewed, and the distribution of the sample is listed in the following Table 2. The table shows that among those 300 individuals, 100 are at lower income, 150 at middle class, 50 at wealthy. Also, 30 of them are in Group A, 200 in Group B, and 70 in Group C. Further, 10 of them are at lower income level and in Group A, and so on. The total sample size is 300.

Using the χ^2 test procedure, we can test on the hypotheses: *H_0: the three income level groups have the same proportion distributions in groups A, B, and C.* V.S. *H_1: the three income level groups have different proportion distributions in groups A, B, and C.*

The numerical procedure to implement the χ^2 test can be found in, for example [1]. By applying that numerical procedure to conduct the χ^2 test, we found that the χ^2 test statistics value equals 28.95. With the degree of freedom (3-1)(3-1) = 4, the corresponding P-value is about 0.000008 which is almost zero. With such a small P-value, we believe that the three income level groups have different proportion distributions in groups A, B, and C. In other words, we believe that in this simulated nation, the impacts of the technology developments on people in the three different income level groups are not equal.

References

[1] C. Brase and C. Brase. *Understandable Statistics,* U.S.A.: Cengage Learning, Boston, 2012.

[2] G. R. Noubary and Y. Shi. "Difference equation models for estimating athletic records", *Journal of Computational and Applied Mathematics,* vol. 91, pp. 107 -122, 1998.

[3] Y. Shi. "A Probability Model for Occurrences of Large Forest Fires", in *Proc. 2011 International Conference on Forestry and Agriculture Engineering and Computer Science,* Wuhan, China, pp. 595 -597, 2011.

[4] Y. Shi. "On the Applications of Markov Chains in Estimating Advertising Effects", in *Proc. 2008 Global Marketing Conference,* Shanghai, China, Session 7.5, pp. 1-8, 2008.

[5] Y. Shi. "A globalization procedure for solving nonlinear systems of equations", *Numerical Algorithms,* vol. 12, pp. 272–286, 1996.