Optimal Solution of Forest Management Under Risk

Soleiman Mohammadi Limaei

Abstract—**This paper investigate the idea and the field of stochastic phenomena in forestry. The text covers forestry decisions and optimal solution of forest management that are based on stochastic phenomena. Optimal decisions in the presence of stochastic timber prices are discussed. Optimal decisions of forest management under risk in even-aged forests and uneven-aged forests are disused and formulated. Stochastic dynamic programming technique is a reliable method for modeling optimization problems in uneven-aged forest that involve risk and uncertainty.**

Index Terms—**Optimal decisions, forest management, risk management, stochastic dynamic programming.**

I. Introduction

The optimal values and decision in forest management may be studied from many different perspectives. Different kinds of constraints, objective functions and risk or uncertainty could be considered. In most cases, the introduction of new considerations affects the optimal decision rules in one way or another. The decisions are usually classified according to whether they are made under conditions of certainty, uncertainty or risk. Certainty is defined as the case when each decision alternative is known to lead invariably to a specific outcome. Risk is the case when each alternative leads to one of a set of possible outcomes and the probabilities of the possible outcomes are known. Uncertainty refers to the situations where there are such alternatives, each having as its consequence a set of possible outcomes, but the probabilities of the possible outcomes are unknown. Depending on the degree of the decision makers knowledge of the outcome, different decision criteria for evaluating and comparing decision alternatives have been suggested [1].

Forestry operations are unique in that revenues are periodic. This means that income may be realized every 20 years and more. Many phenomena could be happen and affect the optimal decision to long term harvesting age. The traditional forest planning, management and economics theory is based on deterministic assumptions. Also today, the deterministic physical and economic environment is the standard assumption in long term and short term modeling in the forest sector. Most concepts, ideas and economic management rules are derived from models based on the assumptions of perfect information concerning future conditions. One reason for the assumption of a deterministic world is that such a world makes it possible to understand and analyze everything in a simple way [2].

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The reason for deterministic assumption is the simplicity of decisions. Timber prices are difficult to predict accurately, since many things may influence the markets. The stumpage price fluctuates over time and it is very difficult to predict it with high accuracy. Therefore we can regard the stumpage price as a stochastic process. Some other phenomena, such as forest growth, also could be stochastic. Price variation is the most important source of risk in forest management [3]. Previous forest management decisions were based on deterministic approach. Risk management in forestry decisions was suggested by Hool, who first used a Markovian framework to analyze the management of even-aged plantations [4]. There are several studies dealt with forestry decision under risk such as: [1], [2], [5]-[10].

Some sources of risk could change the forest production; here they called factors, such as pests and diseases, fire, wind, climate change, etc). Some sources of risk could change the net present values such as price fluctuation and marketing. Uncertainty exists in most of the factors which affect forest management decisions.

Forest owners making harvesting decisions face many uncertain parameters such as price uncertainty, uncertainty about future growth and quality of retained stands and often forestry decisions about the management of the forest have a long time horizon. As a result, the size of the variation in consequences as a proportion of the decision-maker's wealth can be large and the cost of ignoring risk aversion may be high. Moreover, a forest owner's risk aversion may be important in the choice of an optimal rotation strategy [11]. However, it is usually impossible to incorporate uncertainties of all factors into a decision model which describes an actual forest management problem with a reasonable degree of accuracy. This research aims to investigate the idea and the field of stochastic phenomena in forestry. The paper covers forestry decisions and optimal solution of forest management that are based on stochastic phenomena.

II. Production Risk

The expected forest production can reduce due to different kind of environmental risk. Forest production could include seedling production, timber production and residual stands trees.

Seedling survival after reforestation is an important issue for forest landowners. Different risks such as seedling competition, drought and flood, pests and insects could reduce the chance of seedling survival and increase the costs of reforestation. A forest harvester could damage the residual stand. For example, he can broke the residual tree crown and steam, remove bark from the steam and deform the tree. These injuries could introduce insects and diseases, and ultimately reduce the value of forest stand. Careful equipment operation and felling practices can help reduce the damage done to a timber stand during a forest harvesting operation. Ice storms, glaze wild animals, wind fire etc are the some other production risks in forests [12].

III. Stochastic Price

The markets are not easy to predict as it may to be a rather simple and possible completely determined by the parameters of the supply and demand functions. In the forest product market the econometric studies are employed in order to determine the coefficients of supply and demand equations.

Often, very convincing econometric estimates of the parameters of these simultaneous equation systems are reported. The intersections of the equations of supply and demand are calculated and the solutions are compared to the historical price and quantity series. Sometimes, such comparisons look very good. However, these market models contain exogenous variables. Standing timber or stumpage price is dependent on many factors. Prices primarily reflect the current state of the economy, the demand for different species at the time of the sale, the timber quality on the tract, and current market conditions in the area. In forestry and wood products, our markets are closely tied to the housing segment of the economy. As housing starts fluctuate, therefore it will affect the demand for lumber and associated products. When the demand for housing products goes down, the prices paid for standing timber and logs will eventually go down. The threat from substitute products such as plastic and steel also affect lumber prices. As more of these materials are used in home construction, less wood is used, which puts less demand on our forests. Technology influences log and lumber prices as well. As we increase recovery in sawmills with better cutting technologies, we produce more lumber from fewer logs, reducing the demand on our forests. Timber prices directly affect the net revenues of timber harvesting. When timber price is uncertain, the future timber price could be less or higher than the expected price.

The time flexibility of timber growing makes it possible to postpone harvesting when timber price are low. If the timber price is stationary, then one should wait when timber price is low and harvest when it is high. As a result, the existence of timber price uncertainty implies that the timber price at harvesting time is usually higher than the expectations. It is not possible to take advantages when the timber price is non-stationary. Prices are very important to every producer in many ways. In the case of forestry, there are many different prices that strongly affect the firms, directly and indirectly. Let us make some observations:

1. Product price changes (prices of timber, pulpwood, fuel wood etc.) directly influence the profit of the producer in a particular year also if the production (harvest etc.) decisions are held constant.

2. Product price changes indirectly influence the profit in a particular year since they may affect the production (harvest etc.) decisions in that year.

3. Product price changes indirectly influence the profit during future years since those profits are affected by future harvest levels [2].

These future harvest levels are affected by present harvest decisions, which are functions of present prices. There are many empirical studies show that timber price is stationary [3], [13]-[15]. Uncertainty in the forest management costs has the same nature as in timber price. The management costs affect the possible revenues. It is possible to take advantages from management costs fluctuation by adjusting the timber harvest and silvicultural decisions [16].

It is very easy to convince any forest owner without a traditional long term planning education that there is a reservation price. Reservation price can help a forest owner to make the harvest decision. The reservation price for a consumer, it is the maximum value the buyer is willing to pay in order to buy a good. The reservation price for a producer, it is the minimum value the seller is willing to accept in order to sell a good.

The forest owner should harvest, if the timber price is higher than the reservation price. If the price is lower than the reservation price, then he or she should wait at least one more period for a new market price. The forest owner can be indifferent to harvest or wait, if the price is exactly equal the reservation price. Optimal reservation prices in forestry have been calculated by [13], [17]-[20]. Nevertheless, it is not easy to explain how to calculate the optimal reservation price. In order to understand the fundamental principles of such a calculation, a special course is usually needed, including some calculus, stochastic processes, time series analysis and applications of relevance to the issue in question.

IV. Financial Risk

The risk of default is probably the greatest financial risk facing small timberland owners. With the exception of large forest holdings, forestry operations are unique in that revenues are periodic. This means that income may be realized every 15 years, for example, rather than every single year. The expenses associated with holding land and growing stock occur yearly and are generally weighted to the beginning of the rotation/investment cycle (i.e., when the trees were planted or established). These expenses include property taxes, annual management expenses, mortgage interest, and estate taxes on inherited property. Since income from forest operations is periodic, timberland owners must plan to cover expenses in the non-revenue years. In addition, they should set aside some income that can be used to keep the growing stock productive. For example, precommercial thinning, timber stand improvement, and herbicide treatment also referred to as intermediate treatments are forest practices that may be needed to ensure proper stocking levels. These intermediate investments must earn a competitive rate of return. That is, the investment in growing stock should never exceed the expected net income (income left over after costs), including interest. The average annual income after costs will vary from $50 to $100 per acre per year at southern pine stands in USA [12].

Financial risks are minimized when site index, timber prices, and interest rates are combined with annual costs for taxes, forest management practices, and debts to determine the optimum rotation length for a forest stand. In Virginia, the optimum rotation for pine on average sites varies from 30 to 35 years with state of the art management [12].

V. Forest Management and Risk

Forest management is depends on the silvicultural systems. Even-aged and uneven-aged methods are two major silvicultural systems that practiced by foresters. The risk association with these two kinds of methods will be discussed below.

A. Even-Aged Forest Management and Risk

An even-aged silvicultural system is a planned sequence of treatments designed to maintain and regenerate a stand with one age class. We plant the seedling now and we harvest it for example 20 years later. Consequently the plantation and harvesting repeated over the horizon (Fig.1). Even aged timber management is a group of forest management practices employed to achieve a nearly coeval cohort group of forest trees [21].

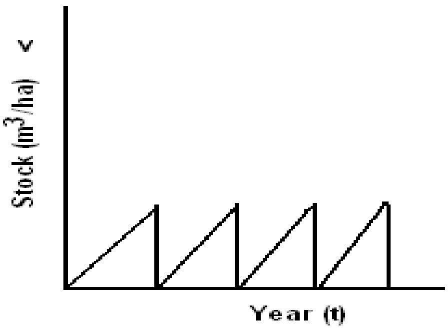

Fig. 1. Cycle of an even-aged silvicultural system.

Even-aged stands cover major part of world forests and the management of such kind of forests is an important subject of forest economics. The aim of forest management is to maximize the NPV. The NPV of forest harvesting function may be written as:

$$Maximize \quad NPV = \sum_{t=0}^{\infty} e^{-rt} R_t \qquad (1)$$

t is the time period.
r is the rate of interest.
R_t is net income.

$R_t = P_t . h_t$

P_t is net price (price - variable harvesting cost).
h_t is harvest level.

In case there are uncertainty about timber price and harvesting costs, then the expected NPV could be written as:

$$\text{Maximize} \quad NPV = E\left[\sum_{t=0}^{\infty} e^{-rt} R_{t\,m}\right] \qquad (2)$$

m is the probability distribution of net timber price fluctuation. Obviously when the timber price is high, the expected NPV is high and vice versa.

If we consider multi period in forestry decision as it is in most real cases in even-aged forests, decisions in different periods should not be analyzed independently. The optimal management alternative for the current period depends on the decisions in the future periods, which, in turn, are affected by the state of nature in the future. If the forest management objective is to maximize the NPV, then the optimal decision for the current period can be determined by the following optimization model:

$$\max_x \quad g(x) + E[W(i_2)] \qquad (3)$$

Subject to $i_2 = f(i_1, x) + \varepsilon_2$

x is the current period decision.
$g(x)$ denotes the NPV associated with choice of x.
i_2 is the state of nature at the next period.
$W(i_2)$ is the maximum NPV when the state at the next period is known.
ε_2 is a series normally distributed errors with mean zero and autocorrelation zero.

In real forest management, the state at the next period is uncertain, the mathematical expectation of $W(i_2)$ is used in the model (3). The future decisions affect the optimal decision for the current period through their impacts on the forest NPV in the next period. Then the following model can be written:

$$W(i_2) = \max \quad E\left[\sum_{m=2}^{n} e^{-r(m-1)} g(\pi_m(i_m))\right] \qquad (4)$$

Subject to $i_m = f(i_{m-1}, \pi_{m-1}(i_{m-1})) + \in_2 \; and \;\; m = 3, 4, ..., n$

Model 4 is calculated to determine the maximum NPV for the next period. Accordingly it is possible to determine the maximum NPV for the future periods with the same procedures.

B. Uneven-Aged Forest Management and Risk

An uneven-aged stand is a group of trees that differ significantly in ages; by convention, the spread of ages exceeds 25% of the planned life span for an age class. In uneven-aged management, mature trees, or groups of them are harvested yearly, leaving gaps and young trees to grow, allocating a portion of the growing space to regeneration. [22]. The development of stock over time is shown in Fig 2. When the forest is harvested, the forest stock or volume per hectare decreased and after lapsing some years the remained trees grows and the forest stock increased. It consist a dentate saw.

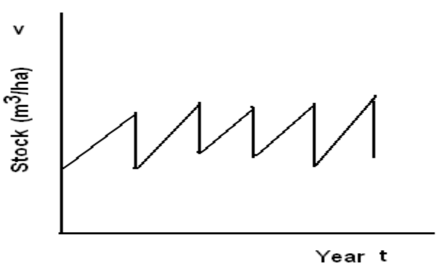

Fig. 2. The relation of stock level and time (year) in an uneven-aged forest.

Harvesting could influence the future stock level and growth in an uneven-aged forest because the future state of nature is as a function of its previous. Hence, we may consider the dynamic property for such kind of forest. The aim of uneven-aged forest management is to maximize the NPV of annual harvesting over the horizon. It could be an easy task to calculate the maximum NPV when we consider that all of phenomena such as price, growth and environment are deterministic. Several past studies of uneven-aged forest management have dealt with the problem of finding the best cutting schedules based on deterministic approaches that maximize NPV such as: [23]-[26].

However, the calculation could be complicated if we consider the development of some important stochastic variables over time and optimize decisions based on the information. Stochastic dynamic programming (SDP) is an acceptable technique for optimization of uneven-aged forest.

The method of SDP was originally presented by [27]. SDP is a useful tool in understanding decision making under uncertainty. The accumulation of capital stock under uncertainty is one example, often it is used by resource economists to analyze bioeconomic problems where the uncertainty enters in such as weather, etc.

Stochastic programming (SD) is a method of modeling optimization problems that involve uncertainty. Deterministic optimization problems are formulated with known parameters. However, in the real world problems almost invariably include some unknown parameters. When the parameters are known only within certain bounds, one approach to tackling such problems is called robust optimization. The goal is to find a solution which is feasible for all such data and optimal in some sense. SD models are similar in style but take advantage of the fact that probability distributions governing the data are known or can be estimated. The goal here is to find some policy that is feasible for all the possible data instances and maximizes the expectation of some function of the decisions and the random variables. More generally, such models are formulated, solved analytically or numerically, and analyzed in order to provide useful information to a decision-maker [28].

As an example, consider two-stage linear programs. The decision maker takes some action in the first stage, after which a random event occurs affecting the outcome of the first-stage decision. A recourse decision can then be made in the second stage that compensates for any bad effects that might have been experienced as a result of the first-stage decision. The optimal policy from such a model is a single first-stage policy and a collection of recourse decisions (a decision rule) defining which second-stage action should be taken in response to each random outcome. Stochastic programming has applications in a broad range of areas ranging from finance to transportation to energy optimization [29].

The qualitative properties of optimal adaptive harvest functions have been determined via analytical SDP by [7], [18]. Optimal adaptive harvest functions with continuous stock and harvest control spaces have been determined by [30].Good descriptions of DP approach and related methods of numerical optimization have given by [31] and [32]. Problems in forestry can be expanded in many different directions. We may consider large numbers of connected decision-making problems. Several of the forestry problems associated with risk and the associated optimization approaches have been described by [2]. SDP method was used to determine the optimal harvest level in uneven-aged forest under stochastic price by [2], [33] and [34].

In order to be able to handle the problem numerically the number of stochastic variables should be reduced and selected for presenting in state space. We must select which variables should be represented in the state space level of detail, the number of different states, or levels, in each dimension.

C. Stochastic Dynamic Programming Formulation

The optimal decisions are determined via SDP approach, in discrete time.

The periods are denoted t. Whereas, $t \in \{0,1,2,...,T\}$ and T is the final period.

$f_t(i)$ is the optimal expected NPV in the beginning of period t when i is the entering state of the system in period t. $R_t(i,j)$ is the profit in period t as a function of the entering state in the same period and the control (j).

$J(i)$ is a set of feasible controls as a function of the entering state, the optimal decisions and expected NPV in final period are determined from:

$$f_t(i) = \max\left[R_T(i,j)\right] \tag{5}$$

were $j \in J(i)$, $i \in I$ and I is the set of states.

The optimal decisions and expected present values in the earlier periods $t, t \in \{0,1,2,...,T-1\}$ are determined recursively via the backward algorithm of SDP:

$$f_t(i) = \max\left\{R_T(i,j) + \frac{1}{1+r}\sum_k p(k|i,j) f_{t+1}(k)\right\} \tag{6}$$

$p(k|i.j)$ is the conditional probability of reaching state k in the next period if your entering state in this period is i and the control is j . r is rate of interest in capital market.

The state space could be two or three dimension. For example one state could be timber price, another state could be stock and the third state could be climate condition or etc. To solve the problem, all of the mentioned state could be stochastic. Price and growth for example, can follow stochastic Markov process or we can reduce the number of stochastic phenomena in order to handle the problem easier.

Finally, the decisions are sequentially optimized based on the latest information concerning the different states.

VI. Jensen's Inequality and Adaptive Production

According to the Jensen's inequality it is possible to take advantages from price variation and adaptive production as producers or adaptive forest harvesting. Fig. 3 shows the general principles.

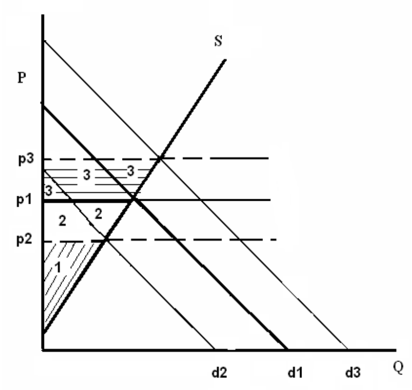

Fig. 3. Demand variations and producer surplus.

If it is assumed that in certainty case the demand function is always at level d1. The supply function is denoted S. The market price is p1. Then, the producer surplus is area 1 plus area 2. Now, we assume that demand is risky and with 50% probability, the demand function is d2 and with 50% probability, the demand function is d3. In the high demand case d3, the price is p3. In the low demand case, the price is p1. In the high demand case d3, the producer surplus is the same as the producer surplus in the certainty case plus the area 3. In the low demand case d2, the producer surplus is the same as the producer surplus in the certainty case minus the surface of area 2. Since the surface of area 3 is larger than the area 2, it is clear that the expected producer surplus is higher in the risk case than in the certainty case. When the price is very low, then the supplied quantity is zero and the producer surplus is still not below zero, according to the Jensen inequality [35].

The inequality states that the convex transformation of a mean is less than or equal to the mean after convex transformation; it is a simple corollary that the opposite is true of concave transformations. The producer surplus increases with increasing price, as long as the price is above the level where the supplied quantity is strictly positive. In addition, the producer surplus increases more and more with increasing prices. The reason is that the price and the quantity increase at the same time. For prices such that the optimal supply volume is strictly positive, the producer surplus is a strictly convex function of price. In such a case, increasing price risk means that the expected producer surplus strictly increases. The expected producer surplus is strictly higher under risk than under certainty in case the relevant prices are found in an interval where the optimal supply is strictly positive, which also means that the producer surplus function is strictly convex.

VII. Conclusion

Forest management is increasingly concerned with understanding effects over long time periods and at broad spatial scales. Prediction of such long-term and broad-scale effects can be very difficult due to the uncertainties associated with more places and longer periods of time can alter outcomes. For example, forest fire is not completely predictable because it depends on future changes in vegetation, fuel and fire spread from many possible ignition sources. Timber prices are difficult to predict accurately, since many things may influence the markets. The stumpage price fluctuates over time and it is very difficult to predict it with high accuracy. Forest growth could be a stochastic phenomenon due to the future change in the environment, tree mortality etc. Therefore we can regard such kind of phenomena as stochastic.

It is explicitly accepted that there are conditions in the environment that can not be perfectly predicted. Harvest revenue can vary widely due to weather, mortality, stock, insects, disease, fire, stumpage price fluctuations and many other factors. In the presence of stochastic phenomena it is important to have many available options. The harvest decision should be taking based on the latest available stumpage price and forest stock information. Actual forest management decisions are never made under conditions of certainty. Deterministic approach more used due to the simplicity in the analysis of forest management problem. Uncertainty in the future state of nature affects the optimal decisions in the current period. Under conditions of uncertainty, silivicultural investment and timber harvest decisions should be made to take advantage of the fluctuations of timber price and management costs, and to reduce the probability of the possible occurrences of undesirable states of nature or their impacts on the outcomes [16]. It is suggested that the mixed species plantation has more advantages than single stands in a stochastic world and could reduce the disadvantages of production and market risks [2].

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