Predicting Financial Failure by Support Vector Machine and Probability of Default of Enterprises in a Developing Country

Bilal Ahmed Khan[1], Longsheng Cheng[1*], Haris Ahmed[2], Muddassar Sarfraz[3]

[1] School of Economics and Management, Nanjing University of Science and Technology, Nanjing, China.
[2] Institute of Business Administration, University of Sindh, Jamshoro 76090, Sindh, Pakistan.
[3] Department of Management and HR, Business School, Hohai University, Nanjing China.

* Corresponding author. Tel.: +86-138-158-73482, email: cheng_longsheng@163.com;
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Abstract: Predicting the financial failure performs an even more significant character in the sustainable existence of enterprises for developing countries. A new risk rating technique based on the probability of default (PD) and order statistics (OS) is established to classify listed companies into two categories according to their financial risks. In the present study, the linear kernel function was united with biorthogonal wavelet kernel function to construct a novel biorthogonal hybrid kernel function. Additionally, the probability of default (PD) and Gray relational analysis (GRA) based new feature weighted approach is established. Grey relational degrees (GRD) between PD and financial indicators are the feature weights (FWOCSVM) on account of that PD can provide effective predicting information for the financial crisis of the listed companies. The financial distress was predicted among financially stable and distressed companies by using feature weighted one-class support vector machine based on the probability of default. The results from collected data of listed companies in Karachi Stock Exchange (KSE), Karachi, Pakistan demonstrated adequate performance by using newly developed approach.

Key words: Probability of default (PD), support vector machine (SVM), order statistics (OS), FWOCSVM, grey relational degrees (GRD), financial failure.

1. Introduction

Corporate failure prediction is a significant management science task and its prediction has a major impact on judgments made by financial bodies, as incorrect judgments can have dreadful financial consequences [1]. All these issues have made predicting enterprise failure one of the key issues which are handled by decision-makers in finance, it leads to the remaking of several financial failure prediction models, and attracting considerable devotion both from businesses and academics [2], [3]. Pakistan is known as a developing country and it is constantly facing business failure. Over the past few years, a large number of bankruptcies were found [4]. Such kind of instability and security threats are tossing several enterprises out of business. A. Rashid and Q. Abbas reviewed an analytical study regarding the business failure of Pakistani companies listed on Karachi stock exchange is still in lack [5]. However, no study was found in this area before their research.

This study proposes a new method for constructing biorthogonal wavelet kernel function further proposes the amalgamation with the other kernel functions method. Basically, the choice of the kernel function and its weight coefficients for optimization are constructed on the basis of this mix-biorthogonal wavelet kernel function. Subsequently, the four dimensions of establishing corporate financial crises early-warning index system of enterprises of different sizes, types, and the industry-based analysis of financial data is proposed which are profitability, solvency, growing capabilities and operational capabilities analysis is based on biorthogonal wavelet kernel for kernel principal component analysis and corporate financial crisis alarm index optimization then finally the hybrid wavelet kernel bases on two level of classifications, multi-value category and another class based on SVM model.

The assessment of upcoming losses for a debtor in credit risk modeling is one the key predictions. The estimation of PD is very important for obtaining a comprehensive credit quality of a debtor. It is better to estimate the probability of default to control the risk of financial failure [6]. The calculation of PD can be carried out by using different techniques. The techniques used for estimation of PD are generally divided into two groups; fundamental based techniques, which normally based on economic factors, systematic market and data of financial statement, and market-based techniques, which is based on ratings and security prices [7]. In the present study, we have used the fundamental-based technique for the estimation of PD.

The present study was conducted to predict financial distress by a novel FW approach based on GRA and PD with a new technique to merge biorthogonal wavelet hybrid kernel and one-class support vector machine (BWH-FWOCSVM). The best part of study emphases on the construction of hybrid kernel function, based on biorthogonal wavelet kernel and the linear kernel. In part 2, the risk rating method based on PD and OS is discussed. The feature weighted approach based on PD and GRA is related in part 3. Part 4 is intended to propose the prediction of financial failure by the feature weighted established on biorthogonal wavelet hybrid kernel one-class support vector machine (BWH-FWOCSVM).

2. Methods

2.1. Risk Rating Method Based on PD and OS

2.1.1. Risk rating method

PD is introduced by KMV model, which is a structural model to assess the credit risk which is proposed by Merton and Black-Scholes as option pricing theory. In the model a company's default market value assets are declined under the book value of liabilities and PD is defined by the SD value of market assets is far from the default point.

PD is calculated as follows in (1):

$$PD = \frac{V_A - DP}{V_A \sigma_A} \tag{1}$$

where V_A denotes the market value of company's assets, σ_A represents the volatility of V_A, and σ_A is the default point. In order to estimate V_A and, we adopt option pricing model proposed by Black and Scholes.

$$V_E = V_A N(d_1) - Xe^{-rt} N(d_2) \tag{2}$$

$$\sigma_E = \frac{V_A}{V_E} \sigma_A N(d_1) \tag{3}$$

In (2) and (3), V_E is the market value of equity, σ_E is the volatility of, X represents book value of debt, and $N(d)$ indicates standard normal cumulative distribution function.

$$DP = SD + \frac{LD}{2} \tag{4}$$

Besides, the default point can be obtained by
where *SD* is the short-term debt and LD is the long-term debt.

2.1.2. Risk rating method

Order Statistics (OS) is a nonparametric method commonly used in mathematical statistics when the overall distribution is unknown beforehand. It has the ability to estimate the overall distribution under a few simple hypotheses [8]. The sample data is ordered by OS, which rearranges the sample data according to their numerical values. After studying the order of the sample data and its properties, the overall distribution can be estimated.

Set the PDs of the companies as the sample (D (1), D (2), ⋯, D(n)), which is arranged in ascending order of magnitude and denoted by (D(1), D(2), ⋯, D(n)). Accordingly, p-quantile of the sample is expressed as

$$\xi_{np} = D_{([np])]} + (n+1)\left(p - \frac{[np]}{n+1}\right)(D_{([np]+1)} - D_{([np])}) \tag{5}$$

Suppose the confidence coefficient is $1-a$ and the corresponding confidence interval is, satisfying

$$P(D_{(r)} > \xi_p) = P(D_{(s)} < \xi_p) = a/2 \tag{6}$$

where $D_{(s)}$ denotes the upper limit and $D_{(r)}$ denotes the lower limit. Thus, r and s are calculated by

$$r = np + 1/2 + u_{a/2}\sqrt{np(1-p)} \tag{7}$$

$$\text{And} \quad s = np + 1/2 - u_{a/2}\sqrt{np(1-p)} \tag{8}$$

When *r* or *s* is not an integer, it will be replaced by the nearest integer.

Based on PD and OS, a new risk rating method is established as follows:

If, then the rating of the corresponding company is T1;

If, then the rating of the corresponding company is T2;

If $D_k \geq D_{(s)}$, then the rating of the corresponding company is T3.

After the above steps, all companies are divided into 3 categories-T1, T2 and T3, representing high risk, moderate risk, and low risk respectively.

2.2. Financial Indicators Selection Based on PD and Grey Relational Analysis

Because of a large number of available indicators in financial reports, choosing the indicators which are utmostly related to company's financial condition is necessary for prediction. In the present study, the choice of financial indicator is grounded on PD and the value of GRD. Grey relational analysis is a method which analyzes the connections amongst series and subsequently describes how each comparison series and series of reference contribute. Different indicators are ranked with GRA by determining the GRD value.

It is used as an extent of the resemblances and significance of different data. The main idea to choose GRA for the present study is to compute GRDs amongst financial condition and the indicators, also to finds out the vital indicators which persuade the most. To calculate the GRD value, we can summarize the GRA method as mentioned below.

GRA analyzes the geometric continuity among dissimilar distinct classifications its immediacy is defined with GRD value, which measures similarities of separated data which could be organized in sequence. In this study, GRA is used for describing correlation among PD and financial indicators which show the relationship of financial condition.

Suppose there are enterprises, having m financial indicators. where $d_0(k)$ denotes the PD for k_{th} enterprises and overall PD values make up the order $D_0 = \{d_0(1), d_0(2), \ldots, d_0(n)\}.x_i(k)$ representing k_{th} enterprises' i_{th} financial indicator, sequthe ence $X_i = \{x_i(1), x_i(2), \ldots, x_i(n)\}(i = 1, 2, \ldots, m)$ comprises of $(k)(k = 1, 2 \ldots n)$. The sequence of the reference is defined with D_0, where (i = 1, 2. . . m) are comparing orders. Grey relational coefficient is used to construct a distinct function for determining the GRD amongst the comparison sequences and reference, which is shown below:

$$\xi_{0i}(k) = \frac{\Delta_{\min} + \rho\Delta_{\max}}{\Delta_{0i}(k) + \rho\Delta_{\max}}, \tag{9}$$

where

$$\Delta_{0i} = |x_i(k) - e_0(k)| \tag{10}$$

is an absolute value variance amongst the i_{th} comparison sequence and reference? $\Delta_{max} = max_i max_k \{\Delta_{0i}\}$ and $\Delta_{\min} = \min_i \min_k \{\Delta_{0i}\}$ are least and highest proximity. $\rho \in (0,1]$ represents distinguish coefficient and $\rho = 0.5$ is usually accepted in many of the previous studies. Hence, the value of GRD amongst the financial situation of the k_{th} enterprise and the i_{th} financial indicator is as shown below.

$$r_{0i} = r(e_0, x_i) = \sum_{k=1}^{n} \omega(k) \cdot \xi_{0i}(k) \tag{11}$$

Here $\omega(k)$ is grey relational (GR) coefficient's weight $\xi_{0i}(k)$, $\omega(k) = 1/n(k = 1, 2, \ldots, n)$ and r_{0i} shows the relation of financial situation with i_{th} financial indicator and it's considered i_{th} financial indicator's weighting.

2.3. PD-FWOCSVM on the Basis of BWHKF

FWSVM is a Support Vector Machine which is created by K_p the feature weighted kernel function:

$$Kp(x_i, x_j) = K(x_i^T P, x_j^T P) \tag{12}$$

Here denotes the kernel function in $X \times X, X \in R^n$ and the feature weighting matrix is $P = diag(r_{01} r_{02}, \ldots r_{0m})$, i is the GRD amongst financial situation and the i_{th} financial indicator, Equation 11 is used to calculate it. i is able to change the geometry of feature space by scaling the geometry of input space as it is considered the weight of the indicator; a better hyper plane is created to improve SVM's classification performance. FWBWHK function based on the PD is presented in (13).

$$K_{MP}(x,x')=\rho\sum_{k\in z}2^{j}(2\pi)^{-2}\int_{-\pi}^{\pi}\phi(\omega)e^{i\omega(2^{j}x^{T}P-k)}d\omega\int_{-\pi}^{\pi}\hat{\phi}(\omega')e^{i\omega'(2^{j}x'^{T}P-k)}d\omega'+(1-\rho)x^{T}PP^{T}x' \qquad (13)$$

By combining inequity of numbers of these to two classes an OC-SVM model is recommended to expand the accuracy rate of classification [9].

Supposing training sample $D=\{x_i,y_i\}\frac{l}{i}=1,$ where the input vector is $x_i\in R^m$, and the class label is $y_i\in\{1,2,\ldots k\}$, which is decided by the risk rating, i.e. T1 shows that the company is in high financial risk and labeled as "1". Therefore, T2 indicates the company has a moderate financial risk and is labeled as "2", while T3 means the low financial risk of the company and is labeled "3".

According to one-versus-all multi-class SVM, the solution of the QPP quadratic programming problem which is equivalent to kth SVM is shown below:

$$\min\frac{1}{2}\|w_k\|^2+c\sum_{i=1}^{l}\xi_i^k s.t y_i^{'}[(w_k,\Phi(x_i))+b]\geq 1-\xi_i^k, i=1,2,\ldots,l\xi_i^k\geq 0, i=1,2,\ldots,l \qquad (14)$$

where $y_i^{'}=1$ if $y_i^{'}=k$ and $y_i^{'}=-1$ otherwise. To deal with this constrained optimization problem, firstly introduce.

Lagrange multiplier, and then take the partial derivatives regarding w_k and ξ_i^k, subsequently arrive at the dual the problem of (21):

$$w(a)=max\sum_{i=1}^{l}a_i-\frac{1}{2}\sum_{i=1}^{l}\sum_{i-1}^{l}a_i a_j y_i y_j(x_i.x_j)s.t\sum_{i=1}^{l}y_i^{'}a_i=0;0\leq C,i=1,\ldots l \qquad (15)$$

$K(x_{j,}x_i)=\left\langle\Phi(x_i)^T,\Phi(x_j)\right\rangle$ is employed to represent the kernel function and the dual problem becomes

$$W(a)=max\sum_{i=1}^{l}a_i-\frac{1}{2}\sum_{i=1}^{l}\sum_{i=1}^{l}a_i\left\langle\Phi(x_i)^T,\Phi(x_j)\right\rangle a_j s.t.\sum_{i=1}^{l}y_i^{'}=a_i=0;0\leq a_i\leq C,i=1,\ldots,l \qquad (16)$$

Through resolving the problem (16), the final decision function $f(x)=\operatorname{sgn}(\sum_{i=1}^{l}a_i^{*}K(x_j,x_i)+b*)$ is acquired. Finally, The kth SVM classifier of the BWH-FWOCSVM model is constructed by substituting (8) into (16). Consequently, the decision function is

$$f(x)=\operatorname{sgn}\left(\sum_{i=1}^{l}a_i*K_{MP}(x,x')-b*\right)=\operatorname{sgn}\left(\sum_{i=1}^{l}a_i*\rho K_1(x^TP,x_i^TP\right)+(1+\rho)K_2(x^TP,x_i^TP)-b*) \qquad (17)$$

2.4. Empirical Experiment

2.4.1. The dataset and indicators

For this study 390 companies are selected from the listed companies of Pakistan Karachi Stock Exchange due to the unavailability of data most of the data is calculated from the company's financial statements which are separately downloadable from KSE website and State Bank of Pakistan. Table 1 shows the category wise indicators used in this paper. There were 25 indicators selected for this study for the period of two consecutive years. The registered companies in KSE, Karachi, Pakistan, are selected as samples to validate the performance of the biorthogonal wavelet hybrid kernels PD-FWOCSVM model in order to predict the financial failure.

Table 1. Categories of Financial Indicators

Category	Financial Indicators
Profitability	Net profit margin, Return on equity, Net profit growth, the Growth rate of equity, the Growth rate of Operating profit, Rate of return on Total Assets, the Growth rate of total assets, Return on assets
Market Value	Tobin's Q, Operating earnings per share, Earning per share, Price-to-sales ratio, Net assets value share, Price to book ratio, Price earnings ratio.
Liquidity	Liquidity ratio, Total assets turnover, Inventory turnover, Current assets turnover, Quick ratio.

According to PD Equation 1-4, the cumulative distribution function of PDs for the samples is achieved and shown in Fig. 1, which indicates that the PDs can be classified by OS.

Half of the samples are used in train model as training samples, whereas, the remaining samples are designated as test samples. In the prediction of business crises and financial distress, the financial indicator is considered as a major foundation by several researchers [10]. Various financial indicators can show changed results of prediction. The commonly used financial indicators are known as earnings per share, current asset ratio, net profit growth rate, inventory turnover, and return on equity [11], [12]. Particularity, in case of Pakistan's accounting system and securities market the financial indicators are normally divided into separate categories, known as market value, debt-paying ability, liquidity and probability and each group holds numerous applicable indicators [13], [14]. In Table 1 four classifications from 25 financial ratios including debt paying, liquidity, market value, and profitability are demonstrated as recorded registered beginning indicators to present the financial condition of companies. The results of GRDs among PDs and financial indicators are shown in Table 2.

Table 2. GRDs between Financial Indicators and PD

	Indicators	GRDs	Indicators	GRDs
$0.9 \leq GRD \leq 1$	EPS	0.989	NAVPS	0.943
	ALR	0.985	TAT	0.924
	LR	0.971	QR	0.917
$0.7 \leq GRD < 0.9$	CAT	0.878	IT	0.793
	PER	0.853	CR	0.776
	ER	0.822	NPGR	0.758
	TQ	0.812		
$0.5 \leq GRD < 0.7$	GRROE	0.674	PSR	0.576
	OPR	0.643	ROE	0.573
	GROP	0.627	NPM	0.567
	DER	0.595	RRTA	0.560
	GRTA	0.595	ROA	0.559
	PBR	0.587	OEPS	0.544

A significant difference was observed in GRDs were 6 GRDs are found higher than 0.9. As shown in Table 2, out of total 25 GRDs, the significant difference in 12 GRDs is between 0.5 to 0.7 while as in 7 GRDs it is between 0.7 to 0.9. Our results showed that the relationship between financial situations and different indicators is quite different. Here, the analogous GRD is considered as financial indicator weight.

3. Results and Discussion

Previous studies used different measurement parameters to predict the financial failure performance of SVM. In the present study the following measurement parameters are considered:

1) *P(D/D):* The %age of Financially distressed enterprises are forecasted as distressed,
2) *P(D/N):* The %age of normal enterprises are forecasted as distressed,
3) CCR: %age of correctly predicted companies from all selected companies.

Model's accuracy prediction is measured by CCR, however, auxiliary discrimination was provided by *P(D/N)* and *P(D/D)*.

It is known that a number penalty costs are resulted due to different types of misclassifications, such as, if a distressed company is classified in the normal company the damage will be higher than misclassification of the normal company in the distressed group. Hence, if different models are found with same or similar CCRs, the higher *P(D/D)* models are preferred.

In the present study, three experimental financial indicator groups are created containing 25 indicators (C_{25}), 13 indicators(C_{13}), and 6 indicators (C_6) corresponding to GRD ≥ 0.5, GRD ≥ 0.7, and GRD ≥ 0.9, respectively. As accthe uracy of prediction in hybrid kernel function is influenced by ρ (weight coefficient), when search step length is 0.1 the analysis of ρ influence was performed by gird search method. Whereas, for ρ =0, liner kernel function and in case of ρ =1, cdf9/7 BW kernel function was used as kernel the function. The results are shown in Table 3, Fig. 2 and Fig. 3. The results are compared with non-weighted PD-OCSVM based on biorthogonal wavelet hybrid kernel function (PD-FWOCSVM). The support vector machine performance is influenced by ε (insensitive parameter) and C (Penalty parameter). Different methods of optimization have been presented, while as PSO (particle swarm optimization) algorithm methods is used in present work to acquire the optimum ε and C.

Table 3 clearly indicates that GRD and PD based weighting of financial indicators can efficiently increase the prediction accuracy. When ρ value is between the range of 0-1 (in 11 cases) the CCRs 1.31%, 1.45%, and 0.84% for PD-FWOCSVM higher than PD-OCSVM, with C_{25}, C_{13}, and C_6, whereas the *P(D/D)s* of PD-FWOCSVM are 2.10%, 2.56%, and 2.33% which are higher than PD-OCSVM, when average value is considered. The comparison of PD-FWOCSVM and PD-OCSVM with maximum CCR was performed when *P(D/N)* and *P(D/D)* were considered for selection criteria. Where both the models having maximum CCR fulfill the given criteria; indicator sets = C_{13}; ρ =0.7. It is clearly shown that PD-FWOCSVM with D/N and D/D 3.2% and 2.62%, respectively are found with more improvement than PD-OCSVM. The model's effectiveness is indicated by D/D to classify the financial failure of companies in precisely on other hand the misclassification of company's financial failure as normal can be resulted as big ria sk. Hence, the effectiveness in predicthe tion of financial failure by using PD-FWOCSVM approach as compared to non-weighted PD-OCSVM is evident.

The relationship between prediction accuracy of a model and financial indicator numbers is not linear, as shown in Fig. 2, 3 and Table 3. In the case of both PD-FWOCSVM and PD-OCSVM, the lowest CCR was observed in C_6, C_{13} is found with highest CCR, while as, uneven CCRs in C_{25} noted. Same was the case with findings of *(D/N)* and *(D/D)*. Our results revealed that very less financial indicators were selected when GRD threshold level is too high which resulted in lots of information and accuracy decline. While as low GRD threshold level raises so many financial indicators that may also effect on accuracy prediction due to information exclusion. The accurate threshold value for financial failure is very important when the financial indicator is dependent on GRD. Actually, the selection of suitable financial indicator group is the key problem.

As demonstrated in Fig. 3 and 4 the CCRs in $C25$, $C13$, or $C6$ of PD-FWOCSVM and PD-OCSVM raise up to 3rd quarter and latterly fall down with the increase of ρ. With the $\rho \in [0.7, 0.8]$ the maximum CCR was observed. The comparison of non-weighted and weighted CCRs between linear kernels (LK), cdf9/7 biorthogonal wavelet kernel (Cdf9/7 BWK) and hybrid kernel average of 9 CCRs is demonstrated as A-BWHK, while as, the maximum CCR of biorthogonal wavelet kernel based SVM is demonstrated as H-BWHK with the change in ρ from 0.1 to 0.9. In $C25$, $C13$, and $C6$ indicator groups for both PD-OCSVM and PD_FWOCSVM, the H-BWHK is with evident improvement and the A-BWHK is greater than CCR of Cdf9/7 BWK and LK. However, the selection of ρ has higher effa ect on accuracy prediction. Improper selection of ρ can be resulted in lower prediction accuracy in case of hybrid kernel function SVM model from single kernel SVM model. Although, optimal algorithm method such as genetic algorithm and PSO and be the substitute of grid search method.

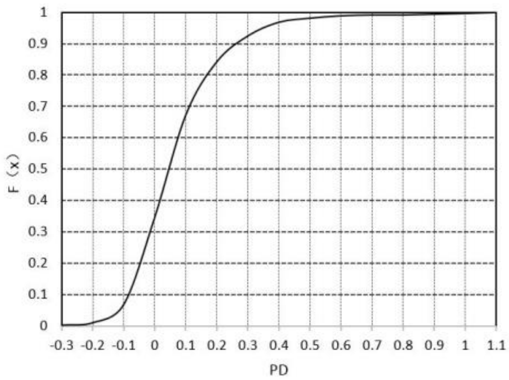


Fig. 1. The cumulative distribution function.

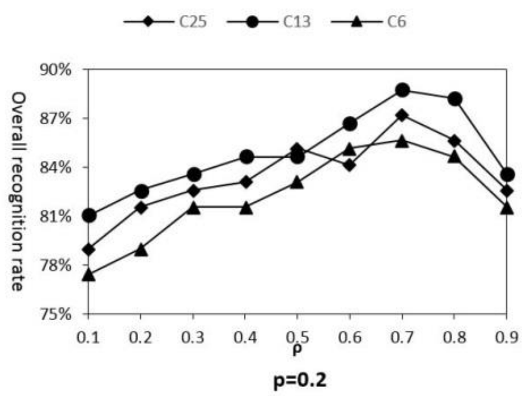


Fig. 2. CCRs of PD- FWOCSVM.

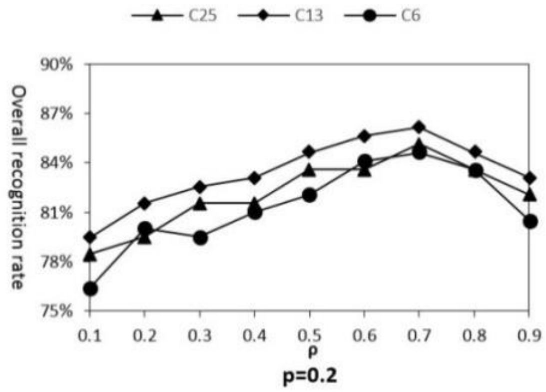
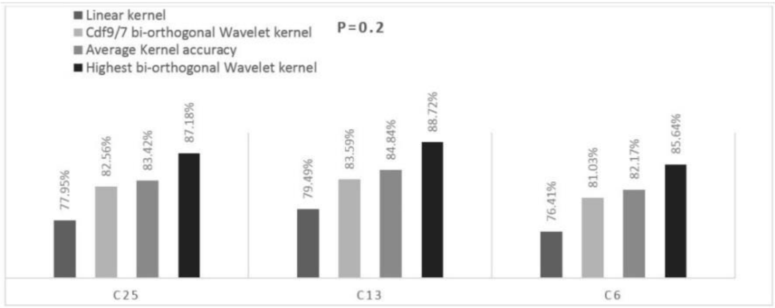


Fig. 3. CCRs of PD-OCSVM.



(a) (b) (c)

Fig. 4. CCRs of PD-FWOCSVM.

Table 3. Results of PD-FWOCSVM and PD-OCSVM (p=0.2)

ρ	Indi	PD-FWOCSVM			PD-OCSVM		
		C_{25}	C_{13}	C_6	C_{25}	C_{13}	C_6
0	CCR	77.95	79.49	76.41	76.92	77.95	75. 38
	P(D/D)	71.79	74.36	66.67	69.23	71.79	64.10
	P(D/N)	20.51	19.23	21.15	21.15	20.51	21.79
0.1	CCR	78.97	81.03	77.44	78.46	79.49	76.41
	P(D/D)	71.79	76.92	69.23	71.79	74.36	66.67
	P(D/N)	19.23	17.95	20.51	19.87	19.23	21.15
0.2	CCR	81.54	82.56	78.97	79.49	81.54	80.00
	P(D/D)	76.92	79.49	74.36	74.36	76.92	74.36
	P(D/N)	17.31	16.67	19.87	19.23	17.31	18.59
0.3	CCR	82.56	83.59	81.54	81.54	82.56	79.49
	P(D/D)	79.49	82.05	76.92	76.92	79.49	74.36
	P(D/N)	16.67	16.03	17.31	17.31	16.67	19.23
0.4	CCR	83.08	84.62	81.54	81.54	83.08	81.03
	P(D/D)	79.49	84.62	79.49	76.92	82.05	76.92
	P(D/N)	16.03	15.38	17.95	17.31	16.67	17.95
0.5	CCR	85.13	84.62	83.08	83.59	84.62	82.05
	P(D/D)	82.05	84.62	82.05	79.49	82.05	79.49
	P(D/N)	14.10	15.38	16.67	15.38	14.74	17.31
0.6	CCR	84.10	86.67	85.13	83.59	85.64	84.10
	P(D/D)	82.05	84.62	84.62	82.05	82.05	82.05
	P(D/N)	15.38	12.82	14.74	16.03	13.46	15.38
0.7	CCR	87.18	88.72	85.64	85.13	86.15	84.62
	P(D/D)	87.18	89.74	84.62	84.62	87.18	82.05
	P(D/N)	12.82	11.54	14.10	14.74	14.10	14.74
0.8	CCR	85.64	88.21	84.62	83.59	84.62	83.59
	P(D/D)	82.05	87.18	79.49	79.49	84.62	76.92
	P(D/N)	13.46	11.54	14.10	15.38	15.38	14.74
0.9	CCR	82.56	83.59	81.54	82.05	83.08	80.51
	P(D/D)	79.49	82.05	76.92	76.92	79.49	74.36
	P(D/N)	16.67	16.03	17.31	16.67	16.03	17.95
1	CCR	82.56	83.59	81.03	81.03	82.05	80.51
	P(D/D)	76.92	79.49	76.92	74.36	76.92	74.36
	P(D/N)	16.03	15.38	17.95	17.31	16.67	17.95

4. Conclusions

In this study, a risk rating method based on the probability of default and order statistics is established to classify listed companies into two ratings according to their financial risks. We developed and implemented a structure of a financial distress prediction model based on publicly available data of listed companies from Pakistan. In conclusion, by the combination of BW kernel function and linear kernel function a unique biorthogonal wavelet kernel function is constructed. We also presented a probability of default and GRA based new feature weight approach, in which the GRDs between PD and financial indicators are the feature weights. On the basis of the new risk rating method and feature weighting, PD-FWOCSVM model classifications are proposed for financial crisis prediction. At last, the analysis of the experiments is conducted based on 390 listed companies in Karachi stock exchange of Pakistan and after analyzing on two different percentile p=0.2 and p=0.3 final figures were compared with other single kernels. Our results showed that the model anticipated in the present study is reliable to evaluate the financial condition of companies.

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Bilal Ahmed Khan received his B.com degree from University of Sindh, Pakistan, in 2008, the MS degree in public administration from University of Sindh, Pakistan, in 2011, and now he is doing his PhD degree in management science and engineering from Nanjing University of Science and Technology, China.

Longsheng Cheng received his BS degree from Anhui Normal University, China, in 1985, the MS degree from East China Institute of Technology, China, in 1988, and his PhD degree in system engineering from Nanjing University of Science and Technology, China, in 1998. From 1998 to 1999, he joined City University of Hong Kong as a research assistant. Since 2005, he has been a professor of management sciences and applied statistics at School of Economics and Management, Nanjing University of Science and Technology. His current research interests include prognostic and health monitoring, machine learning, quality engineering and risk management.

Haris Ahmed Khan received his B.com degree from University of Sindh, Pakistan, in 2016, and now he is doing the MBA degree from University of Sindh, Pakistan.

Muddassar Sarfraz is a postdoctoral fellow at the Department of Management and HR, Business School of Hohai University, Nanjing, PR China. He has obtained the Ph.D. in management sciences and engineering from Business School of Hohai University, Nanjing, PR China. He holds international master of business administration (IMBA) from Chongqing University (China), the master of business administration (HR) from the University of Lahore (Pakistan) and the bachelor in economics from Bahauddin Zakariya University. He has published more than 15 research articles in SSCI, E-SCI-ISI indexed journals and conference proceedings.